Financial Engineering and The Financial Crisis

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About the Financial Crisis, I will not discuss:

- Local politics versus global banks
- Greed, Corporate Governance
- Too big to fail versus too big to save
- Complexity (in various dimensions) and a reigning opaqueness on a massive scale
- The "Heads, the bank wins, tails, you loose"-syndrome, privatizing gains versus socializing losses
- "RM is there to transfer risk from those who do not want to have it to those who understand it" turned out to be a myth

Three early QRM-warnings:

- 1992: Joseph Stiglitz on misunderstanding the power and perceived innovation of loan securitization
- 1998: Embrechts-McNeil-Straumann RiskLab report on properties and pitfalls of linear correlation → our 2005 QRM book
- 2001: RiskLab-LSE report "An academic response to Basel II"

→C.Donnelly, P.Embrechts (2010) The devil is in the tails. ASTIN Bulletin 40(1), 1-33

It Doesn't Take Nostradamus

JOSEPH E. STIGLITZ (1992!)

(Economists' Voice: www.bwpress.com/ev November, 2008)

"I went on to explain how securitization can give rise to perverse incentives …Has the growth in securitization been result of more efficient transactions technologies, or an unfounded reduction in concern about the importance of screening loan applications? … we should at least entertain the possibility that it is the latter rather than the former."

At the very least, the banks have demonstrated an Ignorance of two very basic aspects of risk:
(a) the importance of correlation, and
(b) the possibility of price decline.

Some ingredients of a toxic mix:

- Large Complex Financial Institutions
- Misuse of securitization
- Manufacturing (& holding) of systemic "AAA" tailrisk
- Inadequately capitalized … free lunch!
- Regulatory arbitrage (banking → trading book)
- Some of the LCFIs' warehousing of such risks went from 5 Bio $ in 2/06 to 50+ Bio $ by 9/07
- Leverage: 30+:1
- Accounting misuses: REPO 105, …
- … leading to Wall Street alchemy

In summary (Acharya et al., NYU Stern School, 2010):

"The new banking model of
"originate-distribute-and-hold"
incurred massive systemic tail-risks that
finally brought the financial sector down!"
In other words: these LCFIs were (and
hence(*) the global financial system was)

"long a massive economic catastrophe bond
which was totally mispriced, if priced at all"

* Reasons for "hence": network complexity,
interconnectedness, global business …

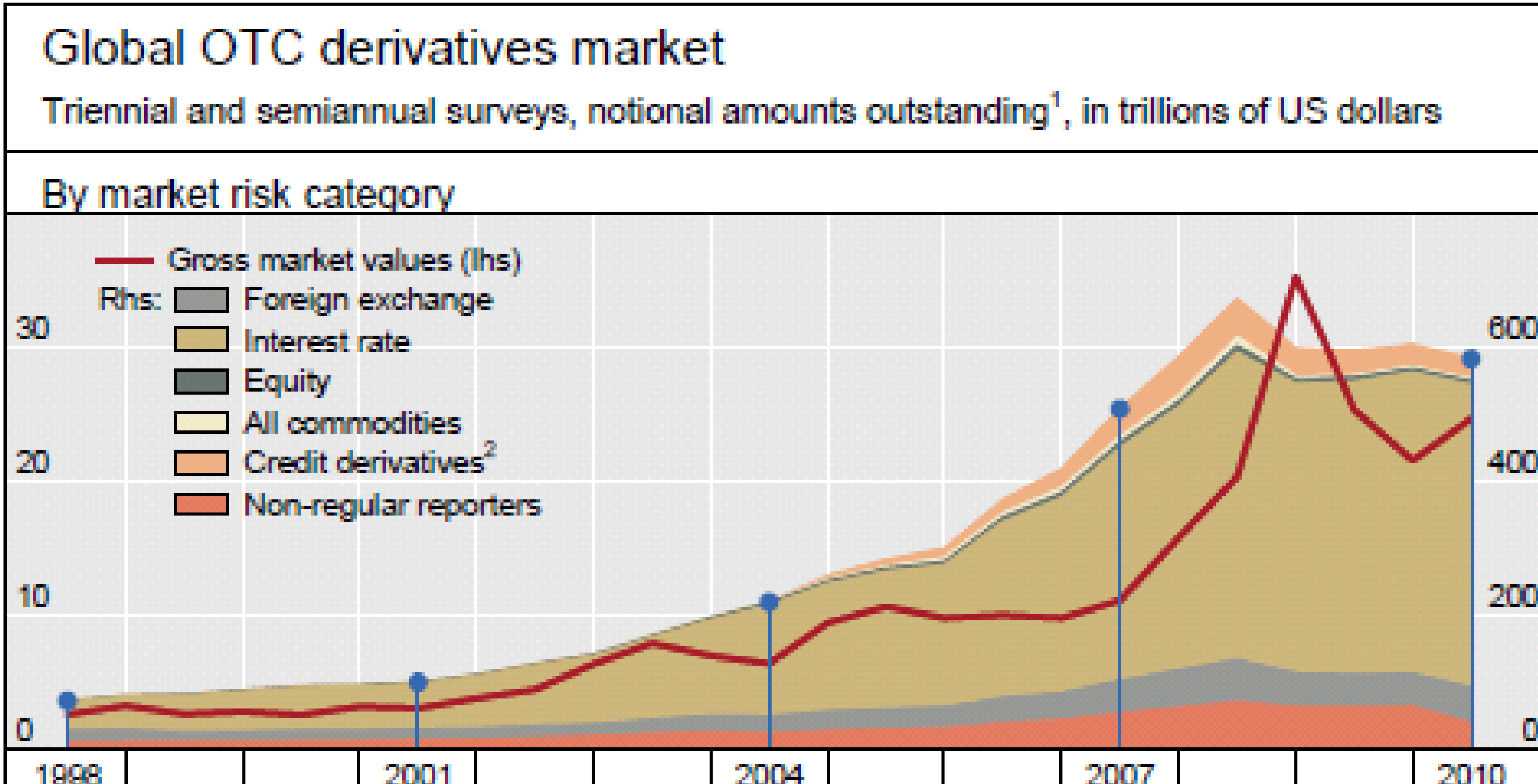
Minimizing the probability of a future crisis with similar devastating consequences:

- Prevention: "RM is most effective at prevention. Failing at prevention results in damage control, which is often expensive and ineffective."
- Education: at all LEVELS, in all FIELDS!!!!
- Communication: we as FE professionals, industry selling products society needs, the media "giving us news we need not just news we want" (Ted Koppel)

Some things we need(ed) to know!

- 1 tri $ = 1 000 000 000 000 $
- World GDP = 58 tri $, US GDP = 14.5 tri $ (US deficit = 1.35 tri $, debt = 13.6 tri $)
- Nominal amount CDS (6/10) = 30 tri $
- Nominal amount of OTC (6/10) = 583 tri $
- CDO volume 2006: 2.7 tri $
- 1/2007: in the US, about 12 AAA-rated companies, and about 65 000 AAA-rated securitization instruments, etc … etc …

From the BIS' Triennial and Semiannual Surveys on Positions in OTC derivatives Markets at end-June, 2010



[1] Dots mark triennial survey dates and data. [2] Data available from end-December 2004.

Source: BIS. Graph 1

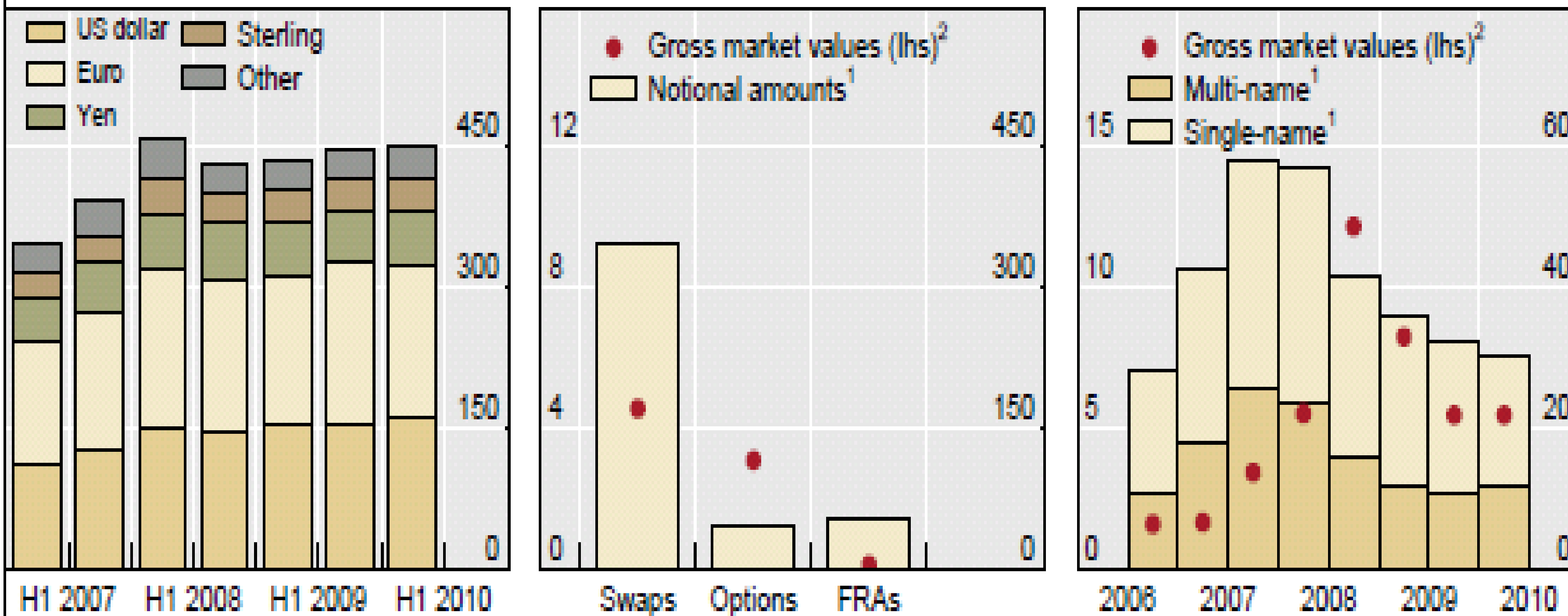
Global OTC derivatives

In trillions of US dollars and in per cent

Interest rate derivatives by currency[1]

Interest rate derivatives by data type and instrument

Credit default swaps by data type and instrument



[1] Notional amounts outstanding (rhs). [2] As a percentage of the notional amount outstanding.

Source: BIS.

Graph 4

Interludium:

- From 1 trillion $ to 1 trillionth of a second! (The latter is called a picosecond (1 ps))
- 1 ps is about the switching time of the (currently) world's fastest transistors
- Light travels 0.3 mm in (+/-) 1 ps
- Quiz: why do I mention this?
- High-frequency trading … do we need it?
- "Speed-of-light trading" … really?
- Co-location (a fact!) etc … what next?

An early warning of things to come?
The Flash-Crash of May 6, 2010!

- 998.50/9.2%
- 600 in 5'

Hence new Risk Management challenges!

Concerning prevention, we tried and failed with:

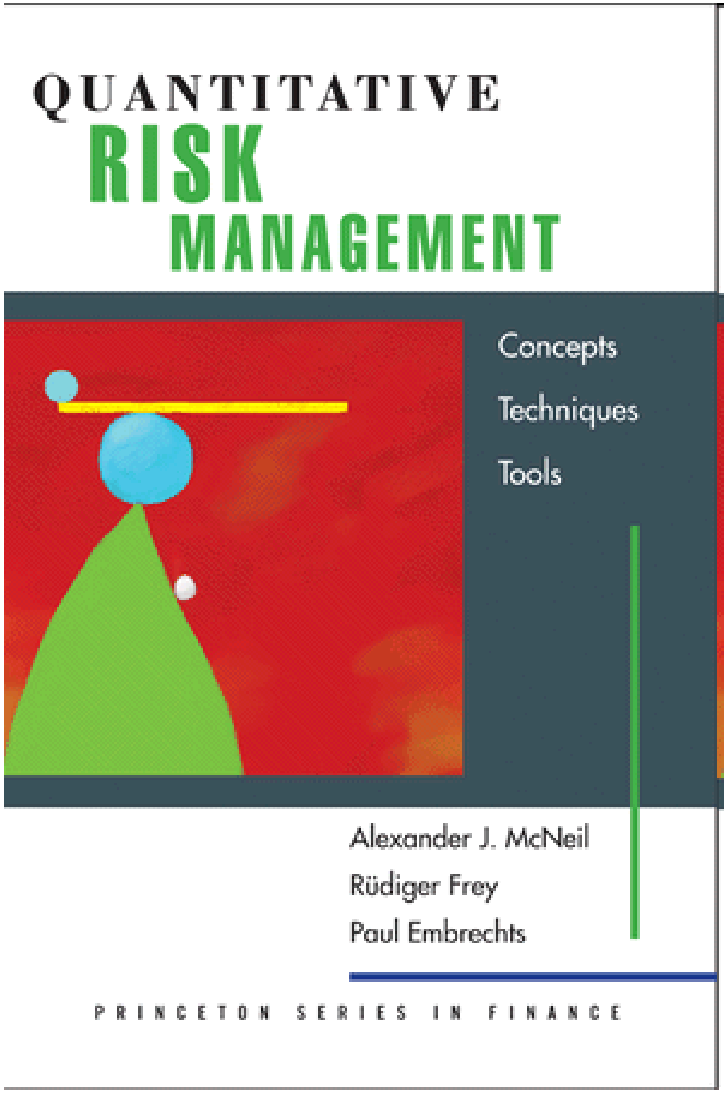
Embrechts, P. et al. (2001):
An academic response to Basel II.
Financial Markets Group, London School of Economics. (Mailed as an official response to the Basel Committee)

→ PE website since 2001!

et al. = Jón Daníelsson
Charles Goodhart
Con Keating
Felix Muennich
Olivier Renault
Hyun Song Shin

In this official response on Basel II we warned very explicitly for:

- Poor quality risk measures (Value-at-Risk)
- Endogeneity of risk, inherent procyclicality
- Lack of measurement of systemic risk
- Impossibility of accurate quantitative measurement of regulatory capital for certain risk classes (OR, 99.9%, 1yr VaR)
- Insufficient quality of rating agencies' assess-ment of default risk for securitized products
- Industry-wide underestimation of downside/ extreme risk, and - dependence ("correlation")



… because of the latter (see also Stiglitz (1992) and Embrechts-McNeil-Straumann (1998)) we included:

Chapter on Extreme Value Theory
"life beyond Normality"

Chapter on Dependence Modelling
"life beyond Linear Correlation"

2005

and much more FE relevant material …

Some FE Examples

- EVT and the POT method
- A note on Risk Measures and an application to the modeling of Operational Risk
- Model Uncertainty (1): micro-correlation and the (mis-)pricing of CDO tranches
- Model Uncertainty (2): a correlation fallacy
- Blame FE (Mathematics)

EVT and the POT method

Some isues:

RM too often frequency oriented ...

- every so often (rare event)
- return period, 1 in x-year event
- Value-at-Risk (VaR)

... rather than more relevant severity orientation

- what if
- loss size given the occurence of a rare event
- Expected Shortfall E[X I X > VaR]

This is not just about theory but a RM attitude!

The Peaks Over Threshold (POT) Method

The excess distribution

Crucial point!

Given that a loss exceeds a *high threshold*, by how much can the threshold be exceeded?

Let u be the high threshold and define the *excess distribution* above the threshold u to have the df

$$F_u(x) = P\left(X - u \leq x \mid X > u\right) = \frac{F(x+u) - F(u)}{1 - F(u)},$$

for $0 \leq x < x_F - u$ where $x_F \leq \infty$ is the right endpoint of F.

Extreme value theory suggests the GPD is a *natural approximation* for this distribution.

Asymptotics of Excess Distribution

Theorem. (Pickands–Balkema–de Haan (1974/75)) We can find a positive, measurable function $\beta(u)$ such that

$$\lim_{u \to x_F} \sup_{0 \le x < x_F - u} \left| F_u(x) - G_{\xi,\beta(u)}(x) \right| = 0 \,,$$

if and only if $F \in \mathrm{MDA}\,(H_\xi)$, $\xi \in \mathbb{R}$.

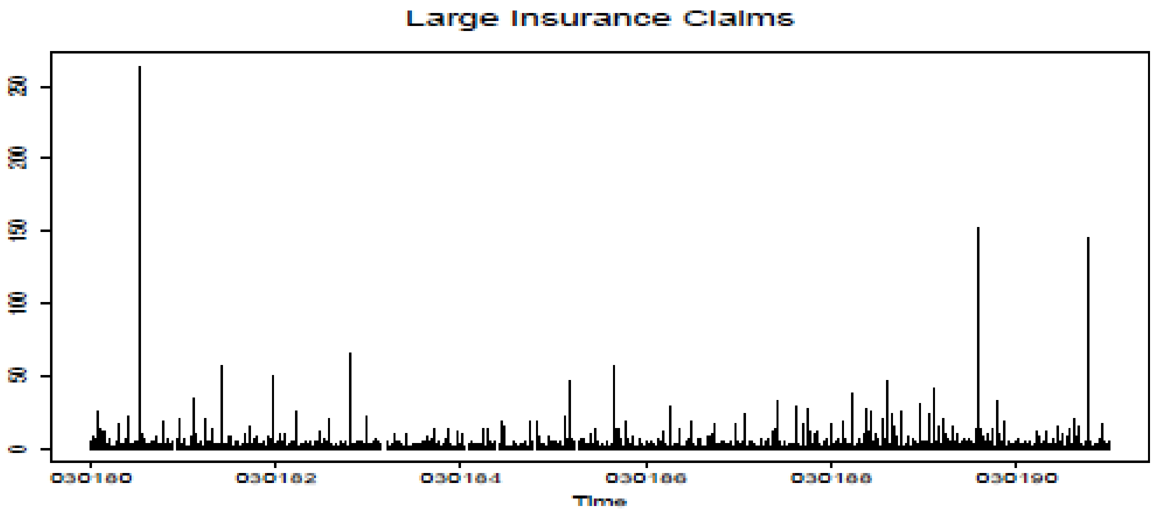
- The GPD is a two parameter distribution with df

$$G_{\xi,\beta}(x) = \begin{cases} 1 - (1 + \xi x/\beta)_+^{-1/\xi} & \xi \neq 0 \,, \\ 1 - \exp(-x/\beta) & \xi = 0 \,, \end{cases}$$

where $\beta > 0$ and $a_+ = \max(a, 0)$, so the support is $x \geq 0$ when $\xi \geq 0$ and $0 \leq x \leq -\beta/\xi$ when $\xi < 0$.

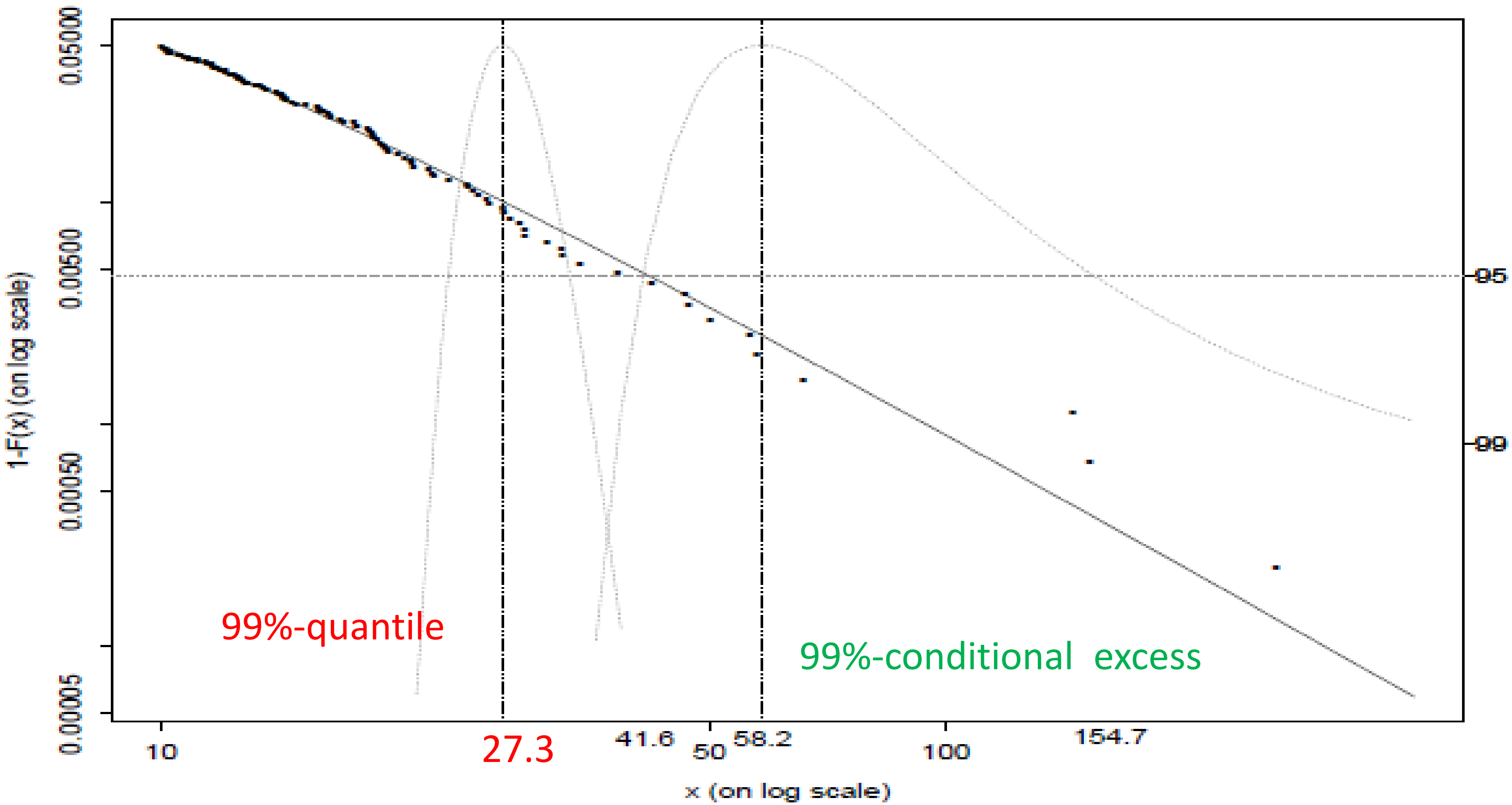
Danish Fire Loss Example

The Danish data consist of 2167 losses exceeding one million Danish Krone from the years 1980 to 1990. The loss figure is a total loss for the event concerned and includes damage to buildings, damage to contents of buildings as well as loss of profits. The data have been adjusted for inflation to reflect 1985 values.



99%-quantile with 95% aCI (Profile Likelihood):
27.3 (23.3, 33.1)

99% Conditional Excess: E(X I X > 27.3) with aCI



Several extensions of 1-d EVT exist:

- Non-stationarity
- Co-variable modelling within POT
- Beware of discrete data, non-standard theory
- Multivariate extremes: definitions

  Several, question dependent approaches exist
- Dynamic, stochastic process models
- Diagnostic and graphical tools
- Important: Communicating extreme events
- Warning: often very slow convergence!!! →

$$\lim_{u \uparrow x_0} \underbrace{\sup_{x \in (0, x_0 - u)} \left| F_u(x) - G_{\xi, \beta(u)}(x) \right|}_{=: d(u)} = 0$$

Rate of convergence to the GPD for different distributions, as a function of the threshold u

Distribution	Parameters	$\overline{F}$	$d(u)$
Exponential(λ)	$\lambda > 0$	$e^{-\lambda x}$	0
Pareto(α)	$\alpha > 0$	$x^{-\alpha}$	0
Double exp. parent		$e^{-e^{x}}$	$O(e^{-u})$
Student t	$\nu > 0$	$\overline{t}_\nu(x)$	$O(\frac{1}{u^2})$
Normal(0, 1)		$\overline{\Phi}(x)$	$O(\frac{1}{u^2})$
Weibull(τ, c)	$\tau \in \mathbb{R}_+ \setminus \{1\}, c > 0$	$e^{-(cx)^\tau}$	$O(\frac{1}{u^\tau})$
Lognormal(μ, σ)	$\mu \in \mathbb{R}, \sigma > 0$	$\overline{\Phi}(\frac{\log x - \mu}{\sigma})$	$O(\frac{1}{\log u})$
Loggamma(γ, α)	$\alpha > 0, \gamma \neq 1$	$\overline{\Gamma}_{\alpha,\gamma}(x)$	$O(\frac{1}{\log u})$
g-and-h	$g, h > 0$	$\overline{\Phi}(k^{-1}(x))$	$O(\frac{1}{\sqrt{\log u}})$

A note on Risk Measures and an application to the modeling of Operational Risk

A note on risk measures

- Axiomatics ---> coherent/convex risk measures
- Example: $q(\alpha,X)$ as a quantile risk measure or return period, $P(X > q(\alpha,X))=1-\alpha$ (α100%-VaR)

  - estimation for α close to 1 ---> EVT
  - nice properties for elliptical models (MVN)
  - cases which are problematic wrt non-convexity,

$$q(\alpha, X+Y) > q(\alpha, X) + q(\alpha,Y),$$

  concern very skew, or very heavy-tailed risks, or risks with special dependence ---> research! (next speaker)

II. Operational Risk

Basel II Definition

The risk of loss resulting from inadequate or failed internal processes, people and systems or from external events. This definition includes legal risk, but excludes strategic and reputational risk.

Examples:

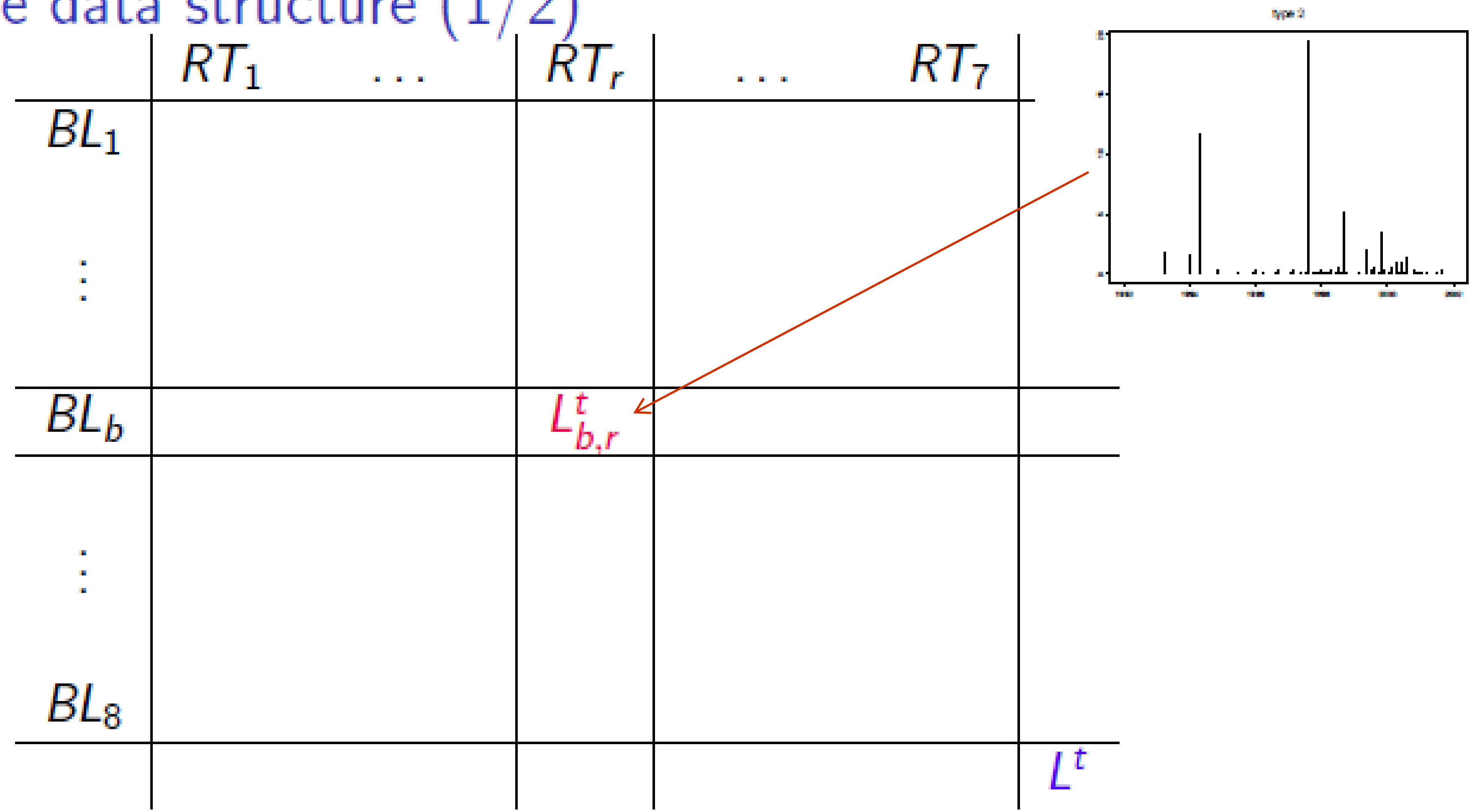
- Barings Bank (1995): $ 1.33 bn (however . . .)
- London Stock Exchange (1997): $ 630 m
- Bank of New York (9/11/2001): $ 242 m
- Société Générale (2008): $7.5 bn

How to measure:

- Value-at-Risk
- 1 year
- 99.9%

} Loss Distribution Approach (LDA)

The data structure (1/2)



$$\mathfrak{X} = \{X_k^{t-i,b,r} : i = 1, \dots, T;\ b = 1, \dots, 8;\ r = 1, \dots, 7;\ k = 1, \dots, N_{b,r}^{t-i}\}$$

$$L^t = \sum_{b=1}^{8} \sum_{r=1}^{7} L^t_{b,r} = \sum_{b=1}^{8} \sum_{r=1}^{7} \left(\sum_{k=1}^{N^t_{b,r}} X_k^{t,b,r} \right)$$

LDA in practice (internal data)

Step 1 Pool the data business-line wise

Step 2 Estimate $\widehat{\text{VaR}}_1, \ldots, \widehat{\text{VaR}}_8$ (99.9%, 1 year)

Step 3 Add (comonotonicity): $\widehat{\text{VaR}}_+ = \sum_{b=1}^{8} \widehat{\text{VaR}}_b$

Step 4 Use diversification argument to report

$$\text{VaR}_{reported} = (1-\delta)\widehat{\text{VaR}}_+, \qquad 0 < \delta < 1$$

(often $\delta \in [0.1, 0.3]$)

Question: What are the statistical issues?

Another talk!

Model Uncertainty (1): micro-correlation and the (mis-)pricing of CDO tranches

A comment on Model Uncertainty:

- X(1), X(2), … , X(d) d one-period risks with P&L distributions F(1),F(2), … ,F(d) (*)
- A financial position Ψ(X(1),X(2), … ,X(d))
- A risk/pricing/valuation/hedging measure R
- Calculate R(Ψ(X(1), … ,X(d))) under (*) and some condition on dependence between the X(i)-positions, i=1,…,d (**)
- Example: calculate VaR(X(1) + … + X(d))

This leads to a Fréchet problem:

- (*) and (**) are typically insufficient for calculating R(Ψ(X(1), … ,X(d))) → MU !!!!
- Remark: (*) and (**) can in general yield no, infinitely many or a unique solution
- At best, one can calculate Upper - and Lower bounds:

$$RL \leq R(\Psi(X(1), \dots ,X(d))) \leq RU$$

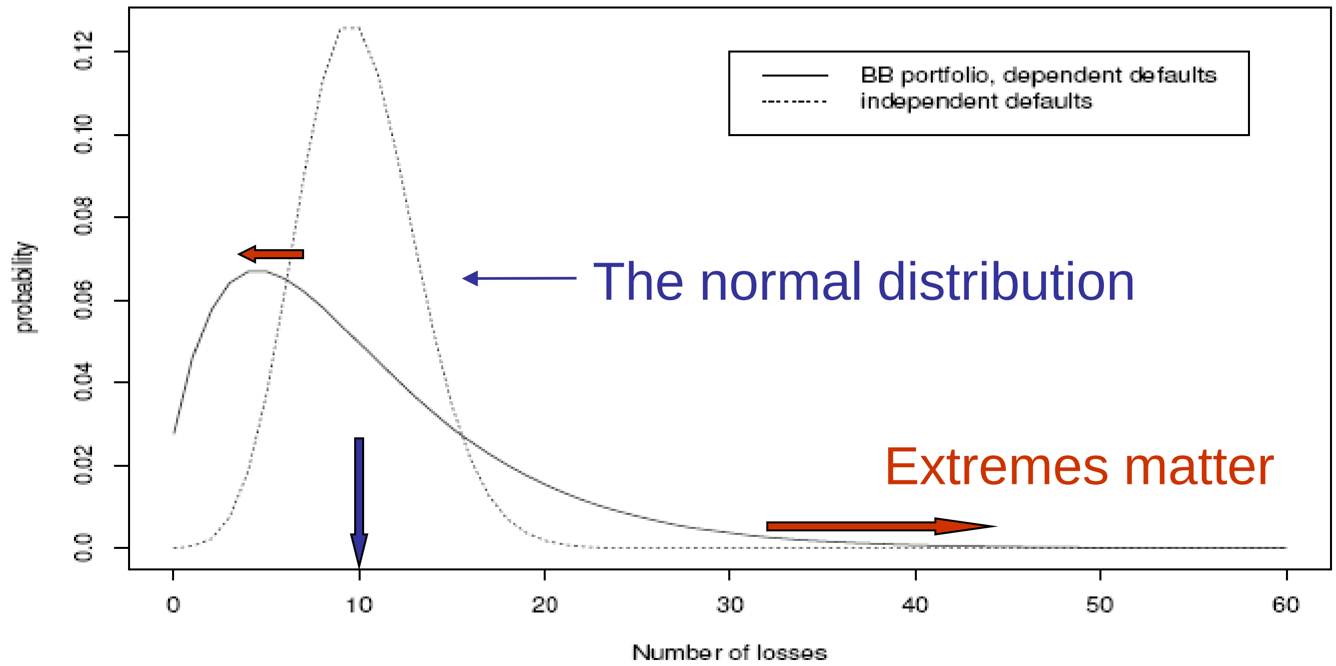
This is without statistical uncertainty!

As an illustration, from Chapter 9, we take the following example, for which the key message is:

beware of (micro-)correlation

micro-

Impact of dependence on loss distribution



Distribution of number of defaults for homogeneous portfolio of 1000 BB loans with default probability $\approx 1\%$; Bernoulli mixture model with default correlation $\approx 0.22\%$ is compared with independent default model.

Correlation matters

As a consequence:

- The pricing (and hedging) of super-senior AAA CDO tranches has substantial model uncertainty (= MU)
- Pricing of CDO**2, CDO**3 products, besides being more than questionable from an economic point of view, is quantitatively near impossible (← MU)
- Hence beware of warehousing such risks!
- Similar examples with other products …

And as a further illustration,
from Chapter 5
Beware of Model Uncertainty (2):
a correlation fallacy

Simulation of a two-dimensional portfolio with marginal distributions given as

F(1)=LN(0,1) and F(2)=LN(0,9)

and dependence:

Corr = 50% → no solution

Corr = 30% → no solution

Corr = 10% → infinitely many solutions

So understand the model conditions!

From the QRM book:

Theorem 5.25 (attainable correlations). *Let (X_1, X_2) be a random vector with finite-variance marginal dfs F_1 and F_2 and an unspecified joint df; assume also that $\operatorname{var}(X_1) > 0$ and $\operatorname{var}(X_2) > 0$. The following statements hold.*

(1) *The attainable correlations form a closed interval $[\rho_{\min}, \rho_{\max}]$ with $\rho_{\min} < 0 < \rho_{\max}$.*

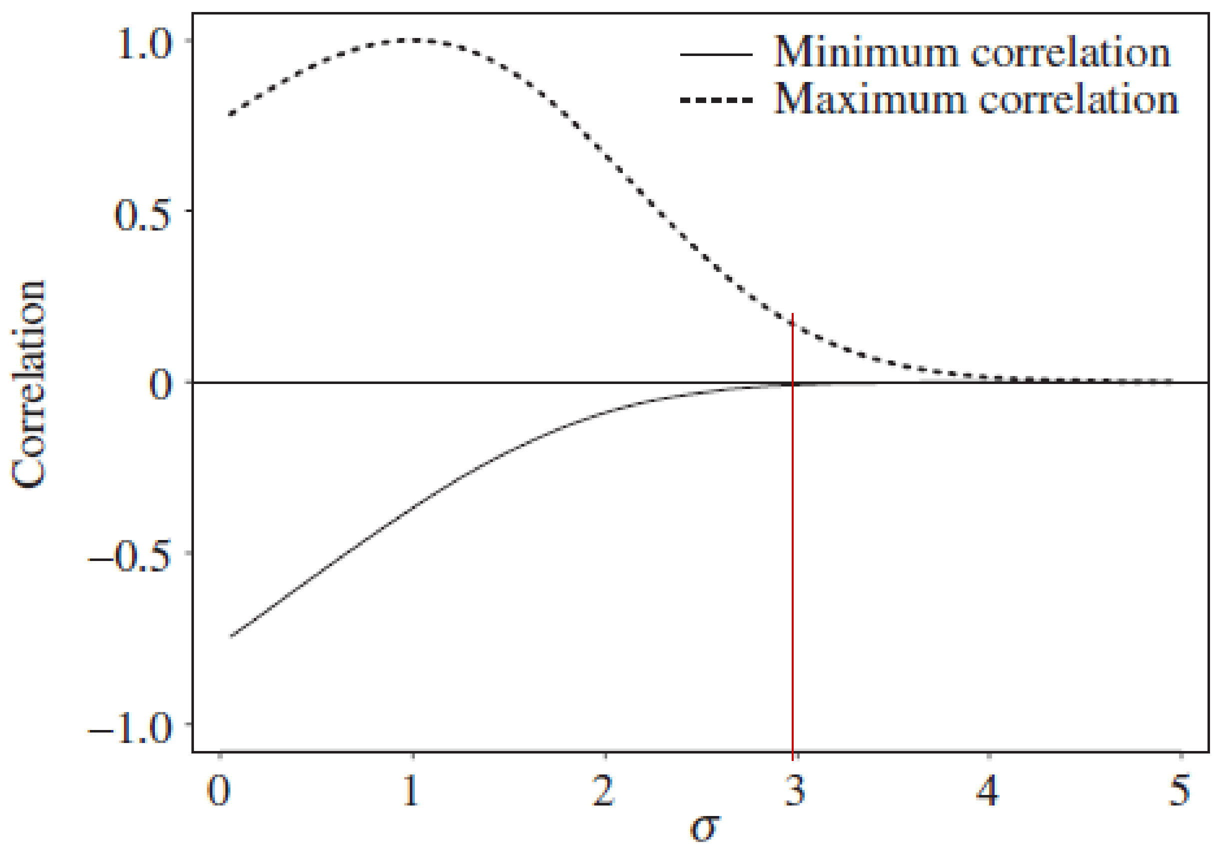
(2) *The minimum correlation $\rho = \rho_{\min}$ is attained if and only if X_1 and X_2 are countermonotonic. The maximum correlation $\rho = \rho_{\max}$ is attained if and only if X_1 and X_2 are comonotonic.*

(3) *$\rho_{\min} = -1$ if and only if X_1 and $-X_2$ are of the same type (see Section A.1.1), and $\rho_{\max} = 1$ if and only if X_1 and X_2 are of the same type.*

(A result due to M. Fréchet and W. Hoeffding (1940s))

As an example: LN(0,1), LN(0,σ^2)

$$\rho_{\min} = \frac{e^{-\sigma} - 1}{\sqrt{(e-1)(e^{\sigma^2} - 1)}}, \qquad \rho_{\max} = \frac{e^{\sigma} - 1}{\sqrt{(e-1)(e^{\sigma^2} - 1)}}$$



Some key MU-issues:

- How to combine marginal risk information into a multivariate model environment
- Copula methodology is one possibility in the static case, however
- Three reasons for using copulas: pedagogic, pedagogic, stress testing
- MU often exists at the structural parametric level (as above) and this on top of statistical (estimation) uncertainty
- OR-Robust Optimisation, ...

Blame FE (Mathematics)

Recipe for Disaster: The Formula That Killed Wall Street

By Felix Salmon 23 February, 2009

Wired Magazine

$$\Pr[T_A<1, T_B<1] = \Phi_2(\phi^{-1}(F_A(1)), \phi^{-1}(F_B(1), \gamma)$$

Error,)

Even the Financial Times joins in:

Of couples and copulas by Sam Jones (April 24, 2009)

In the autumn of 1987, the man who would become the world's most influential actuary landed in Canada on a flight from China. He could apply the broken hearts maths to broken companies.

Li, it seemed, had found the final piece of a riskma-nagement jigsaw that banks had been slowly piecing together since quants arrived on Wall Street.

Why did no one notice the formula's Achilles heel?

Johnny Cash and June Carter

Some personal recollections on the issue:

28 March 1999

Columbia-JAFEE Conference on the Mathematics of Finance,
Columbia University, New York.

10:00-10:45 P. EMBRECHTS (ETH, Zurich):

"Insurance Analytics:
Actuarial Tools in Financial Risk-Management“

Why relevant?

1. Paper: **P. Embrechts, A. McNeil, D. Straumann (1999)**
Correlation and Dependence in Risk Management: Properties and Pitfalls. Preprint RiskLab/ETH Zürich.

2. Coffee break: discussion with David Li.

Two results from the 1998 RiskLab report

CORRELATION AND DEPENDENCE IN RISK MANAGEMENT: PROPERTIES AND PITFALLS

PAUL EMBRECHTS, ALEXANDER MCNEIL, AND DANIEL STRAUMANN

Remark 1: See Figure 1 next page

A very early warning!

Remark 2: In the above paper it is shown that

Thus the Gaussian copula gives asymptotic independence, provided that $\rho < 1$. Regardless of how high a correlation we choose, if we go far enough into the tail, extreme events appear to occur independently in each margin. See Sibuya (1961) or Resnick (1987), Chapter 5, for alternative demonstrations of this fact.

1959

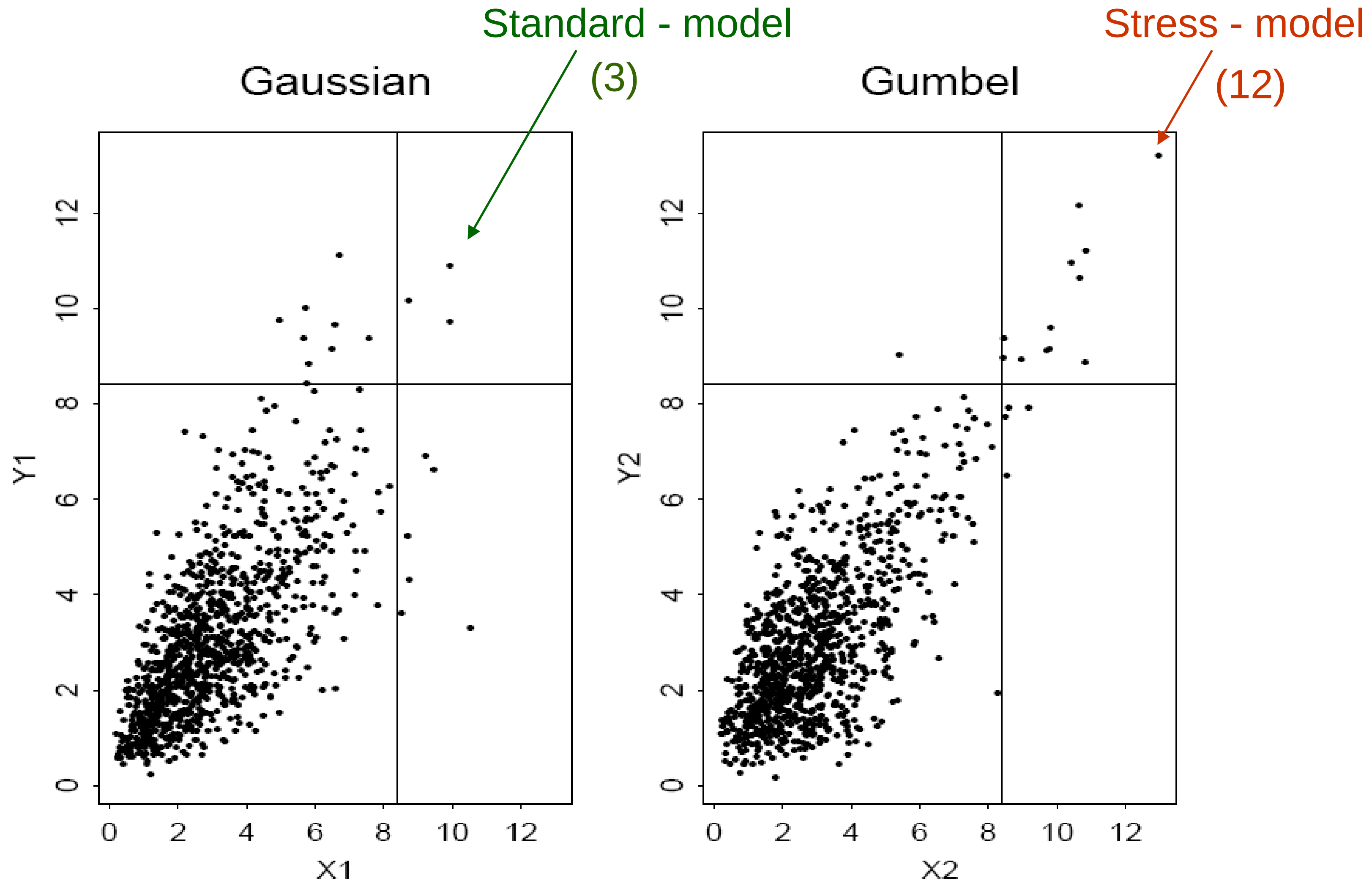


FIGURE 1. 1000 random variates from two distributions with identical Gamma(3,1) marginal distributions and identical correlation $\rho = 0.7$, but *different* dependence structures.

Dear Sir

The article "Of couples and copulas", published on 24 April 2009, suggests that David Li's formula is to blame for the current financial crisis. For me, this is akin to blaming Einstein's $E=mc^2$ formula for the destruction wreaked by the atomic bomb.

Feeling like a risk manager whose protestations of imminent danger were ignored, I wish to make clear that many well-respected academics have pointed out the limitations of the mathematical tools used in the finance industry, including Li's formula. However, these warnings were either ignored or dismissed with a desultory response: "It's academic".

We hope that we are listened to in the future, rather than being made a convenient scapegoat.

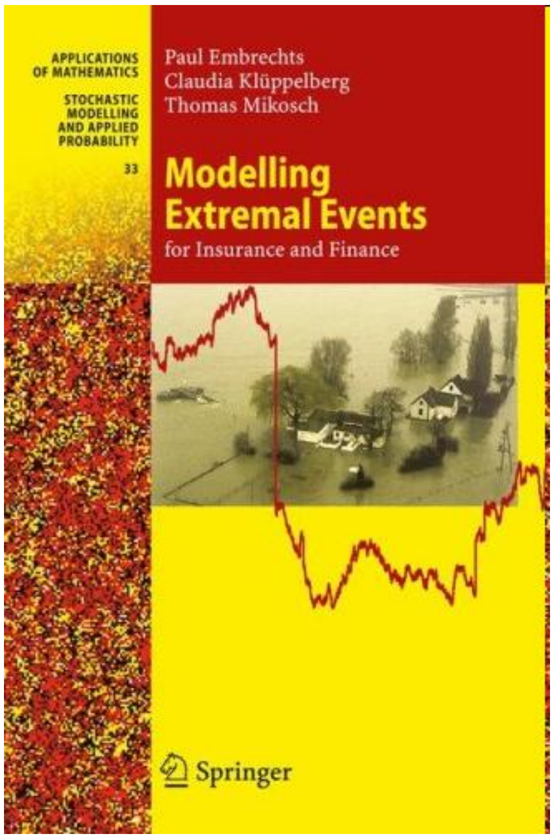
Yours Faithfully,
Professor Paul Embrechts
Director of RiskLab
ETH Zurich

Mathematics is of key importance for

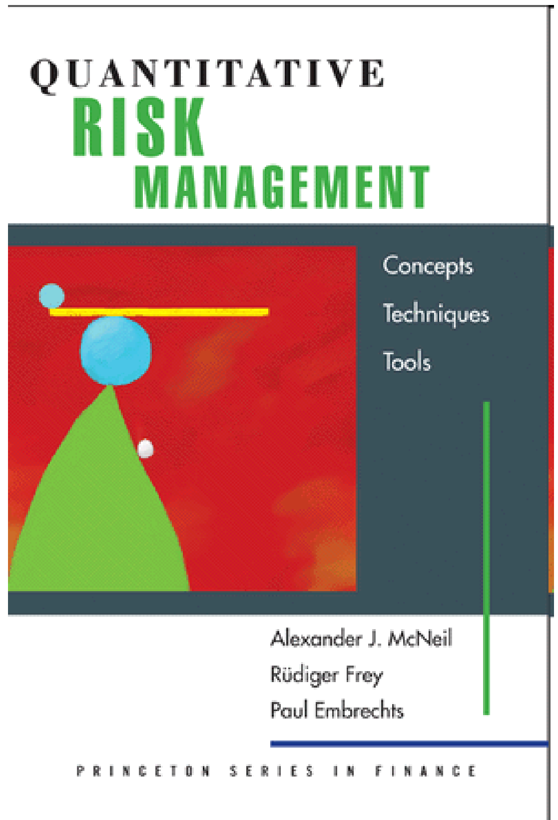
- understanding and clarifying models and prices used in finance, insurance and economics
- making heuristic methods mathematically precise, and asking for clear, unambiguous definitions!
- highlighting model conditions and restrictions on applicability
- working out numerous explicit examples
- leading the way for stress testing and robustness properties
- and it would be bad if the current crisis would induce a shying away from mathematics!

RiskLab QRM Research - Examples (1997, 2005, 2007)

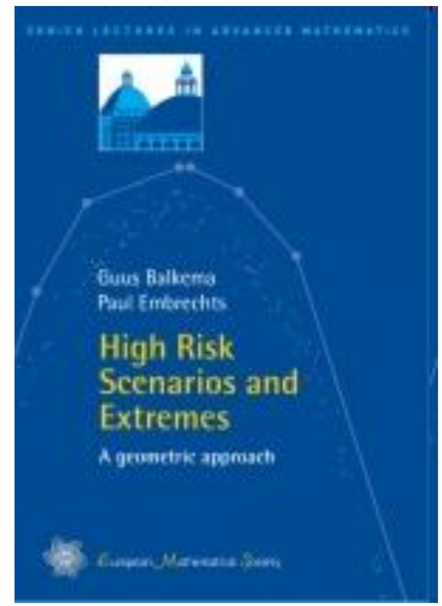
1997



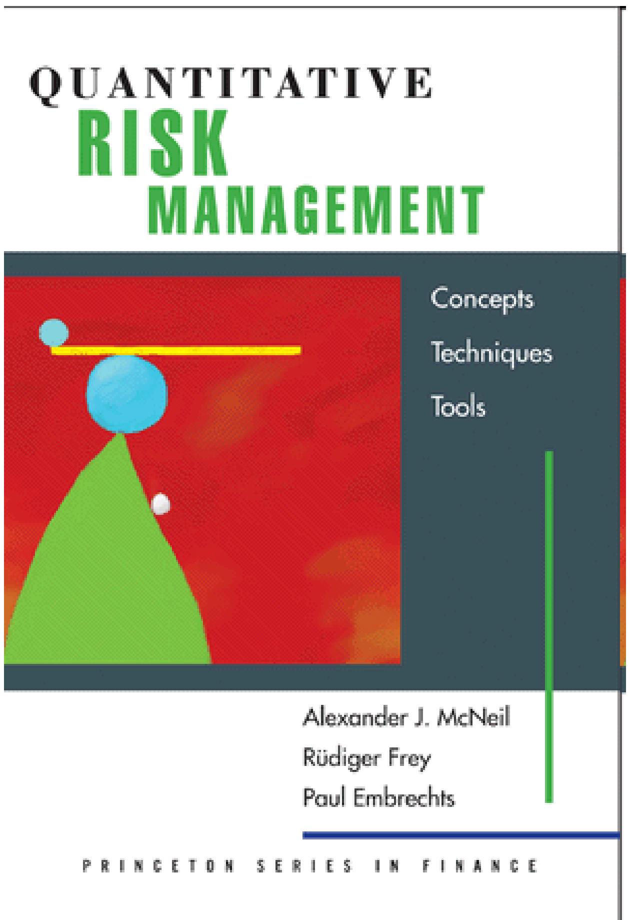
2005



2007

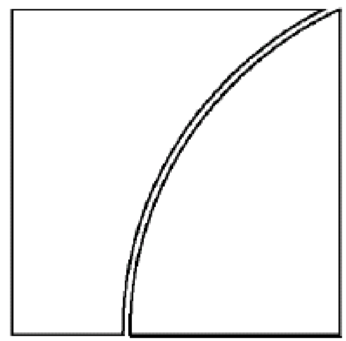


A nice example of the importance of Financial Engineering (QRM) and the research done at RiskLab (relevance!) of ETH Zurich!



Basel Committee
on Banking Supervision

Joint Forum



Developments in Modelling Risk Aggregation

October 2010

BANK FOR INTERNATIONAL SETTLEMENTS

But mathematics is just one small piece of the complex RM puzzle:

Some very basic RM rules:

- If you don’t understand it, don’t sell/buy it
- Speak to “the guys in the boiler room”
- Beware of “new” paradigms, like the New Economy, the New Risk Management: “new” usually means that tried and trusted measures of the past are being ignored
- Always understand your gains and beware of volume (even/especially AAA)
- Concerning Basel II+ or III: do not try to reinvent the wheel, check countries and institutions that came through the crisis less harmed, understand why!!!

Thank you!