The Dilemma of Being Optimally Distinct from Others

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Chair of Decision Theory and Behavioral Game Theory

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ETH Zürich

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Overview

- Provide an introduction to the problem
- Develop a model for this class of games
- Describe normative solutions
- Show simulations from a game theoretic model (with Karsten Donnay)
- Discuss extensions and experiments

I n d i v i d u a l i t y

Always remember that you are unique. Just like everybody else.

UNIQUE

JUST BECAUSE YOU ARE UNIQUE DOES NOT MEAN YOU ARE USEFUL

Optimal distinctiveness

- On being the same and different at the same time (Brewer, 1991; 1993)
- Game theory problem because everybody is trying to do the same thing, which is defined from what everybody does
- And everybody knows that everybody else is trying to do the same, and so on...

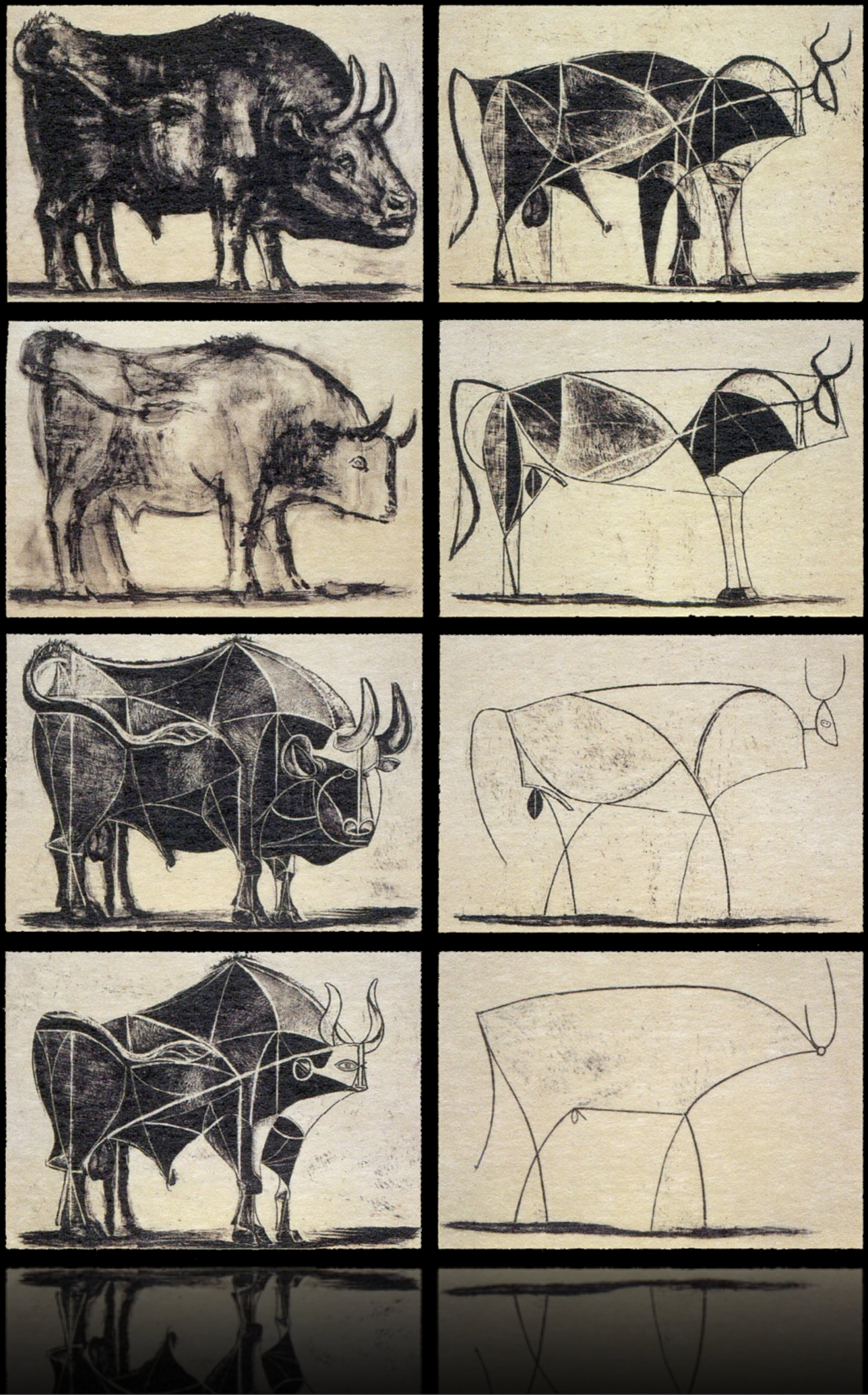
An example

- Consider an example from industrial design
- The average car is generally judged by consumers as unattractive
- Designers have an incentive to develop a car that looks different from all the other cars, but not too different
- But the "average car" is the result of all of these choices, hence the dilemma

Landwehr et al., 2011

An example

- Optimal distinctiveness is a problem for decision makers in a variety of settings
  - Social identity (Brewer, 1991, 1993)
  - Social norms (Hornsey & Hogg, 1999)
  - Industrial design (Landwehr et al., 2011)
  - Markets (Lancaster, 1975)
  - Innovation and leadership (Guastello & Guastello, 1988)



Model building

The goal is to take a decision problem from the messy real world and distill it into something as simple as possible, but no simpler, in such a way that it still retains the central essence of the original problem.

We strive to develop a decision context that is tractable but non-trivial.

Spacial Norms Game

- Consider N (N >> 2) decision makers (players)
- Each player chooses a point in a well defined Cartesian plane (e.g. square from 0 to 100 on each side)
- Players' choices are made simultaneously and privately
- A center point is computed from the N chosen points
- The payoff for each player is a function of the distance from their chosen point to the center point
- The structure of the game and the payoff function are all common knowledge

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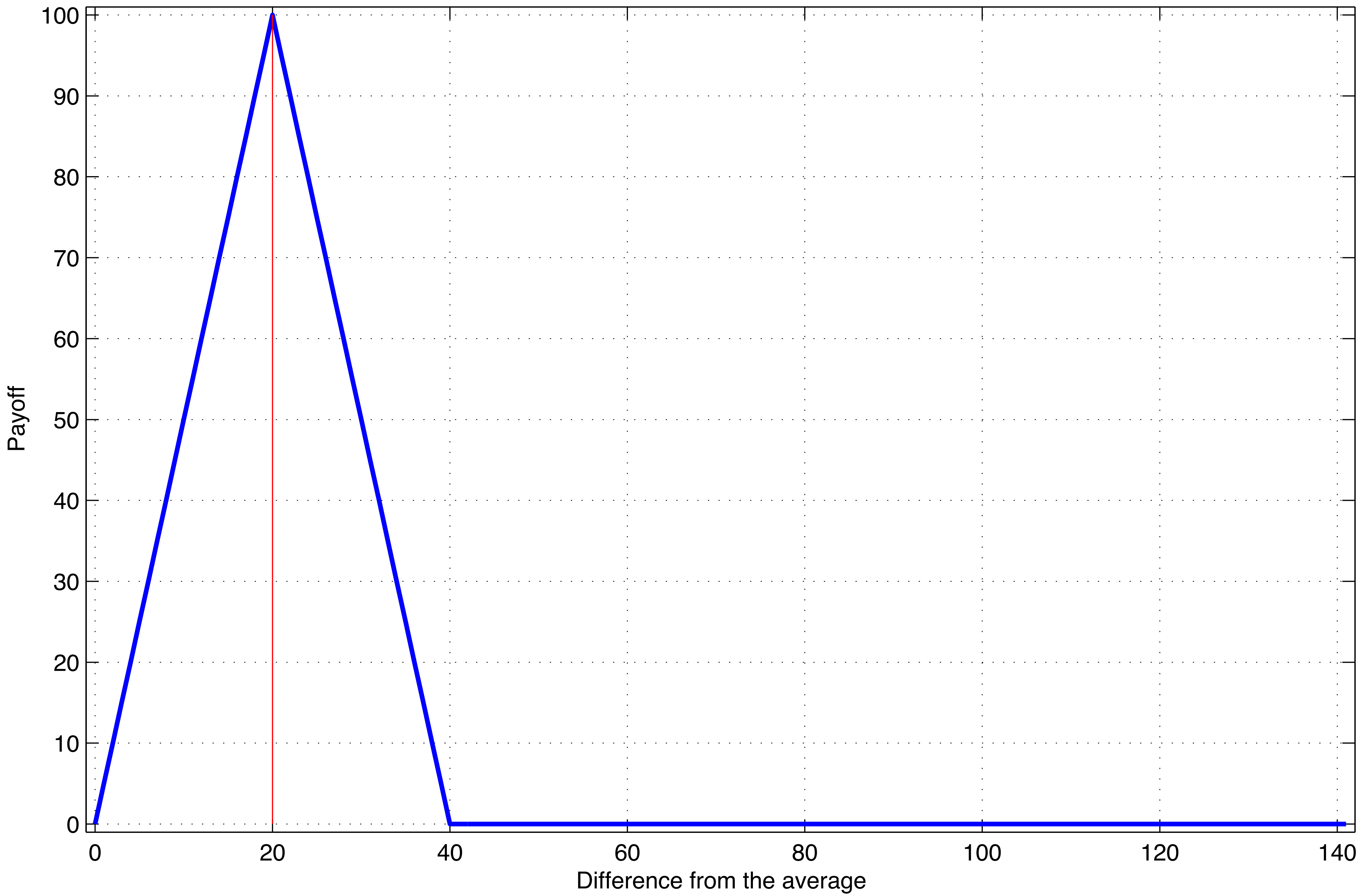
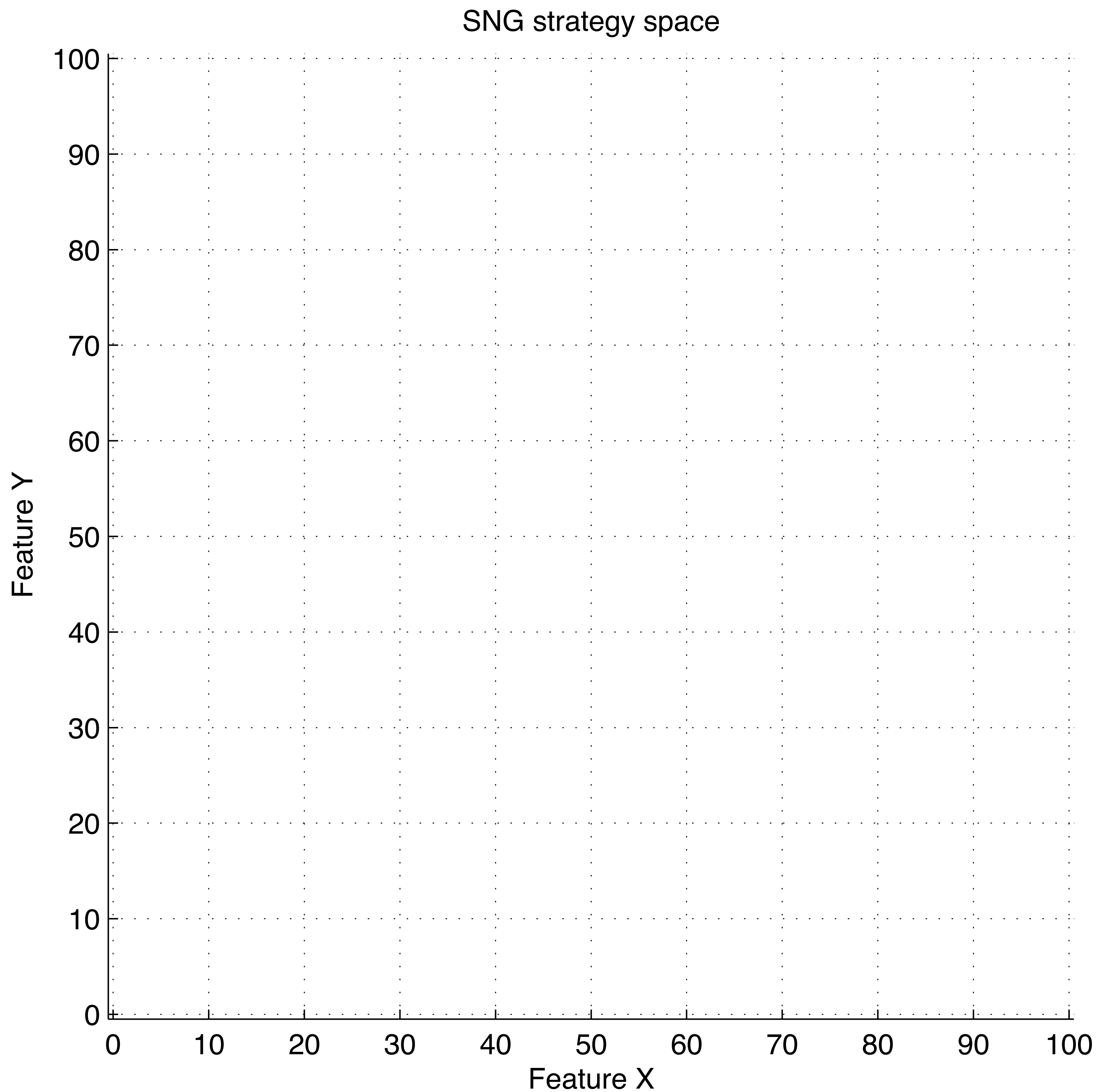
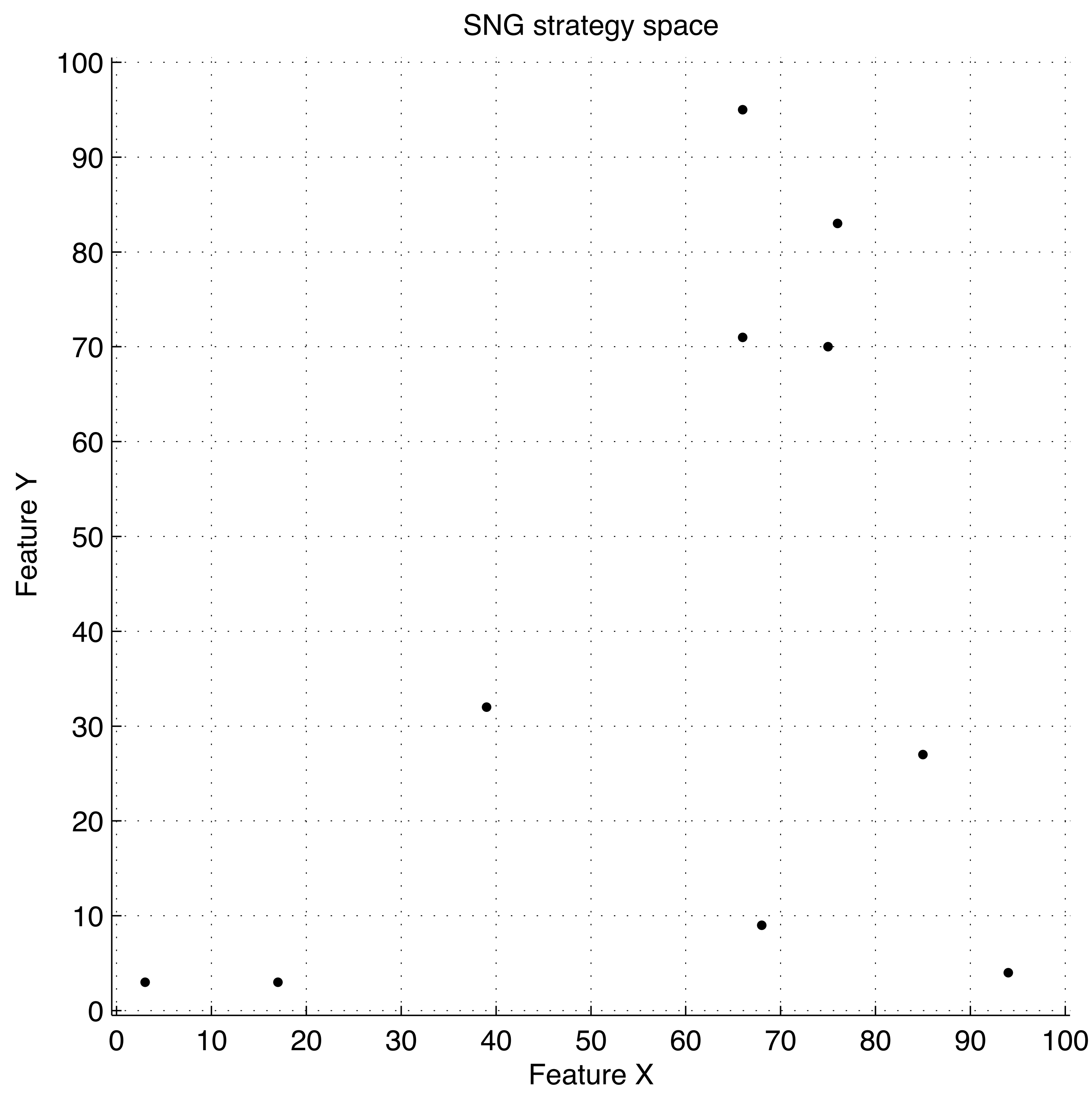
Optimal distinctiveness payoff function

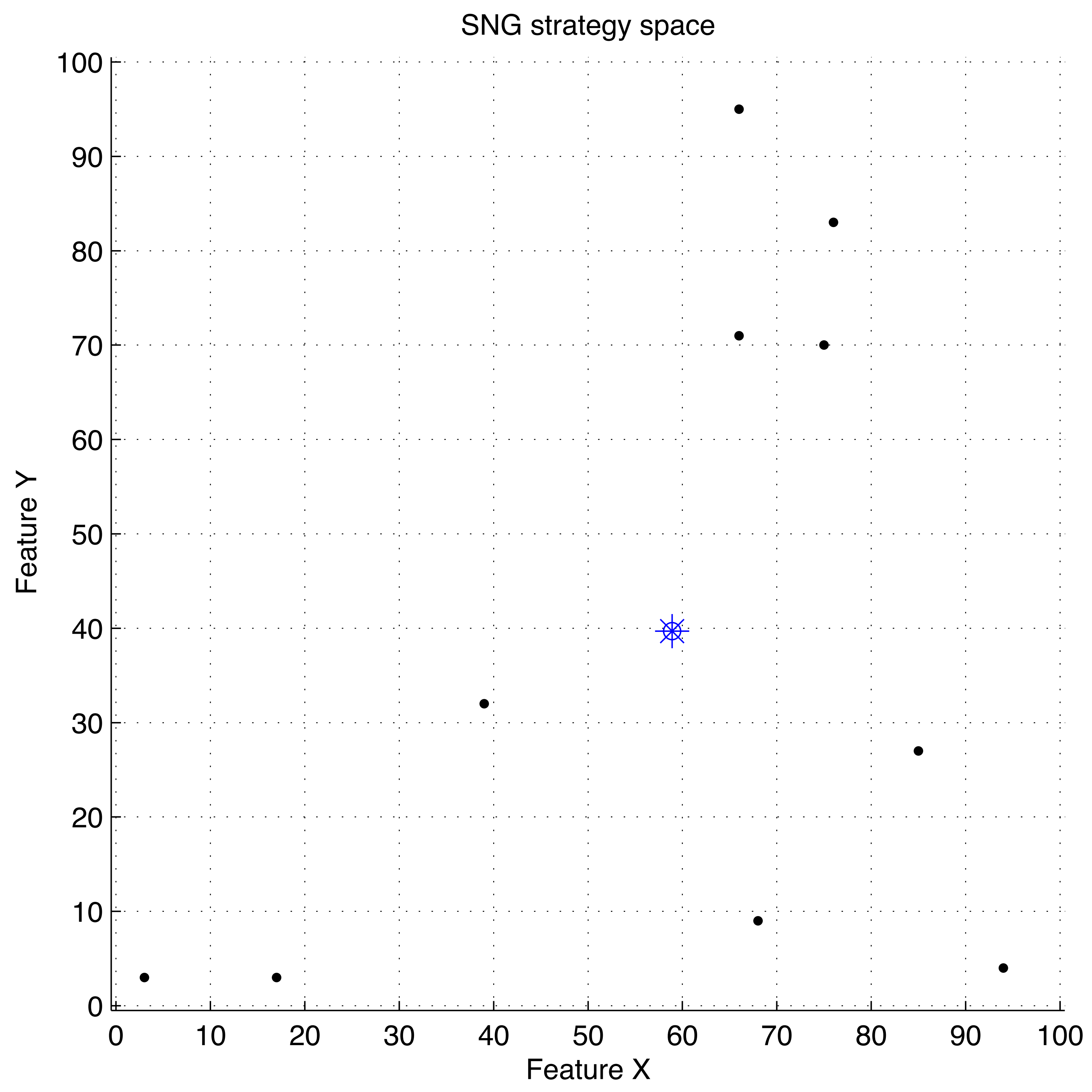




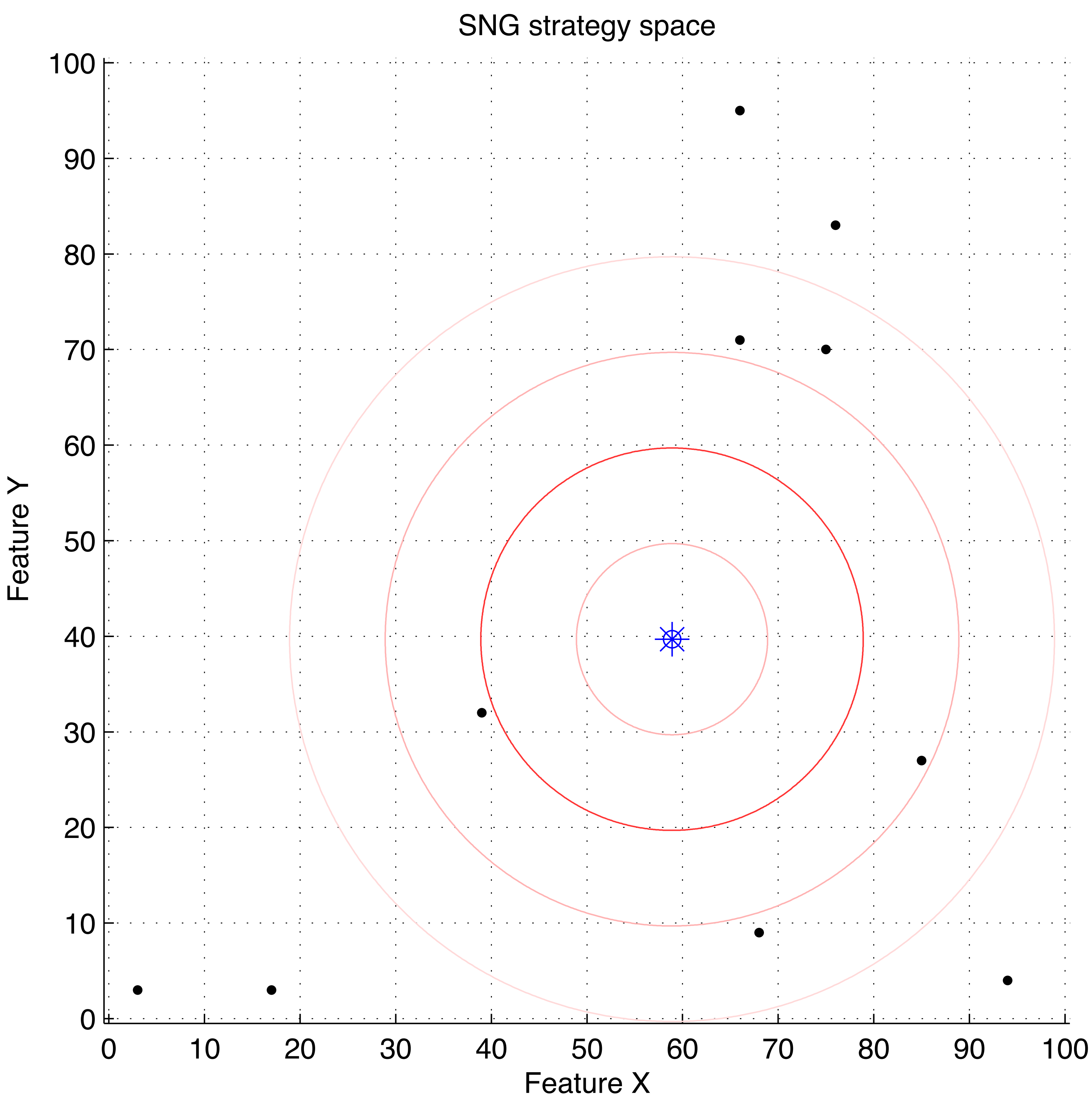
Feature space

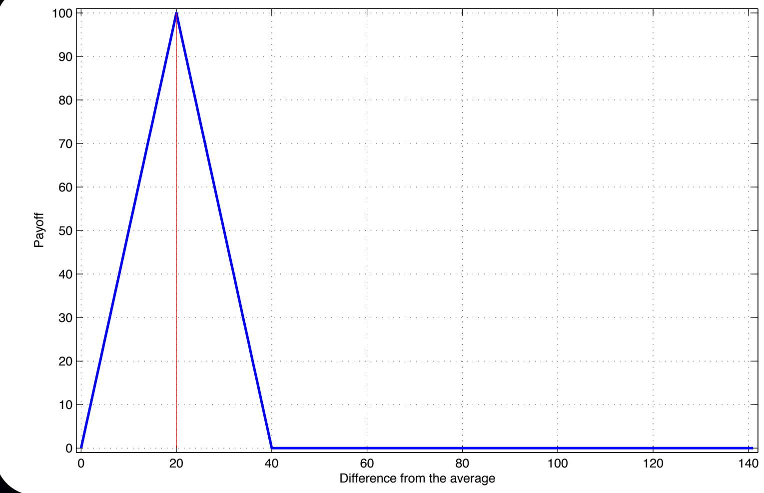


Feature space and choices

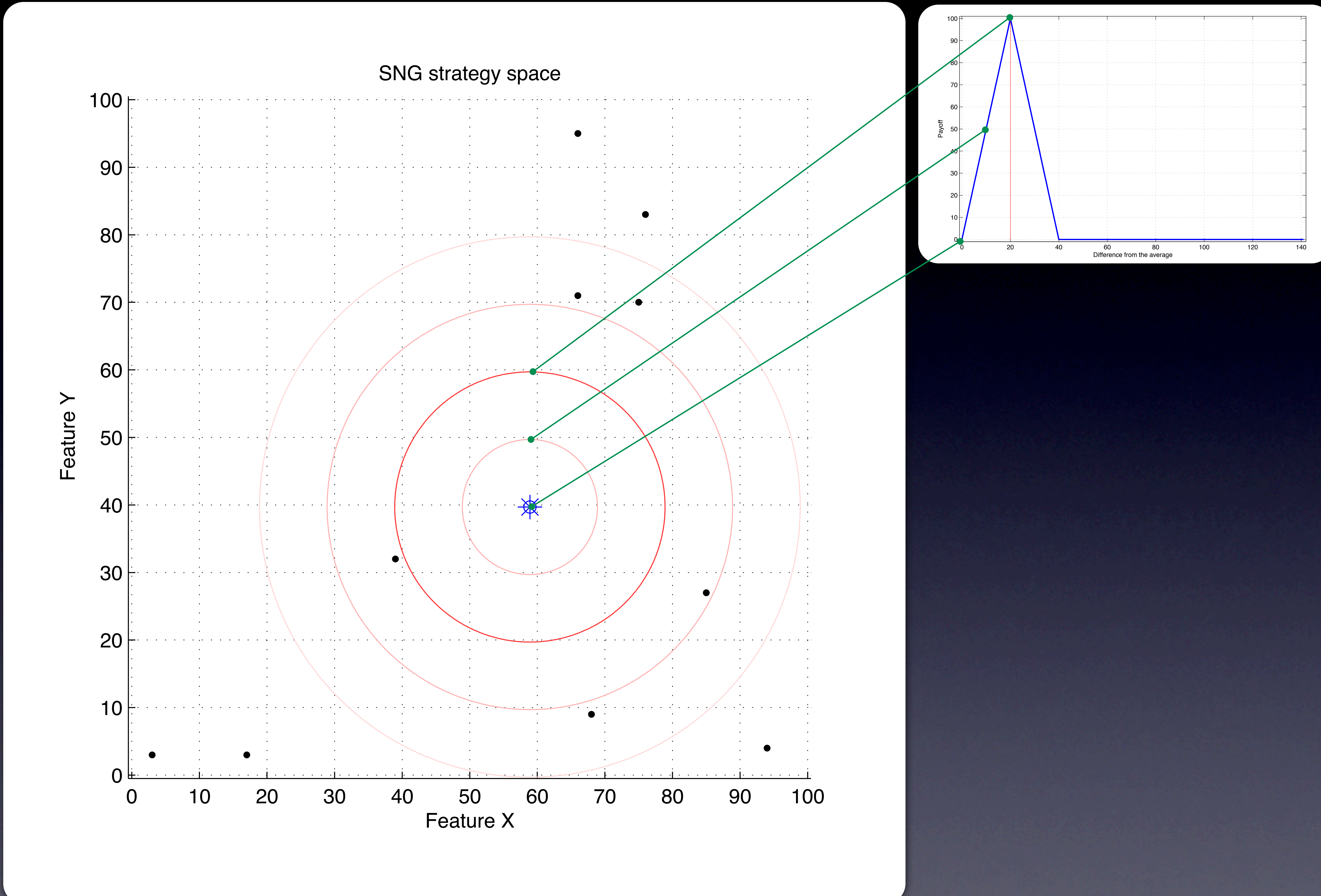


Feature space, choices and center point





Payoffs contingent on what everyone chooses



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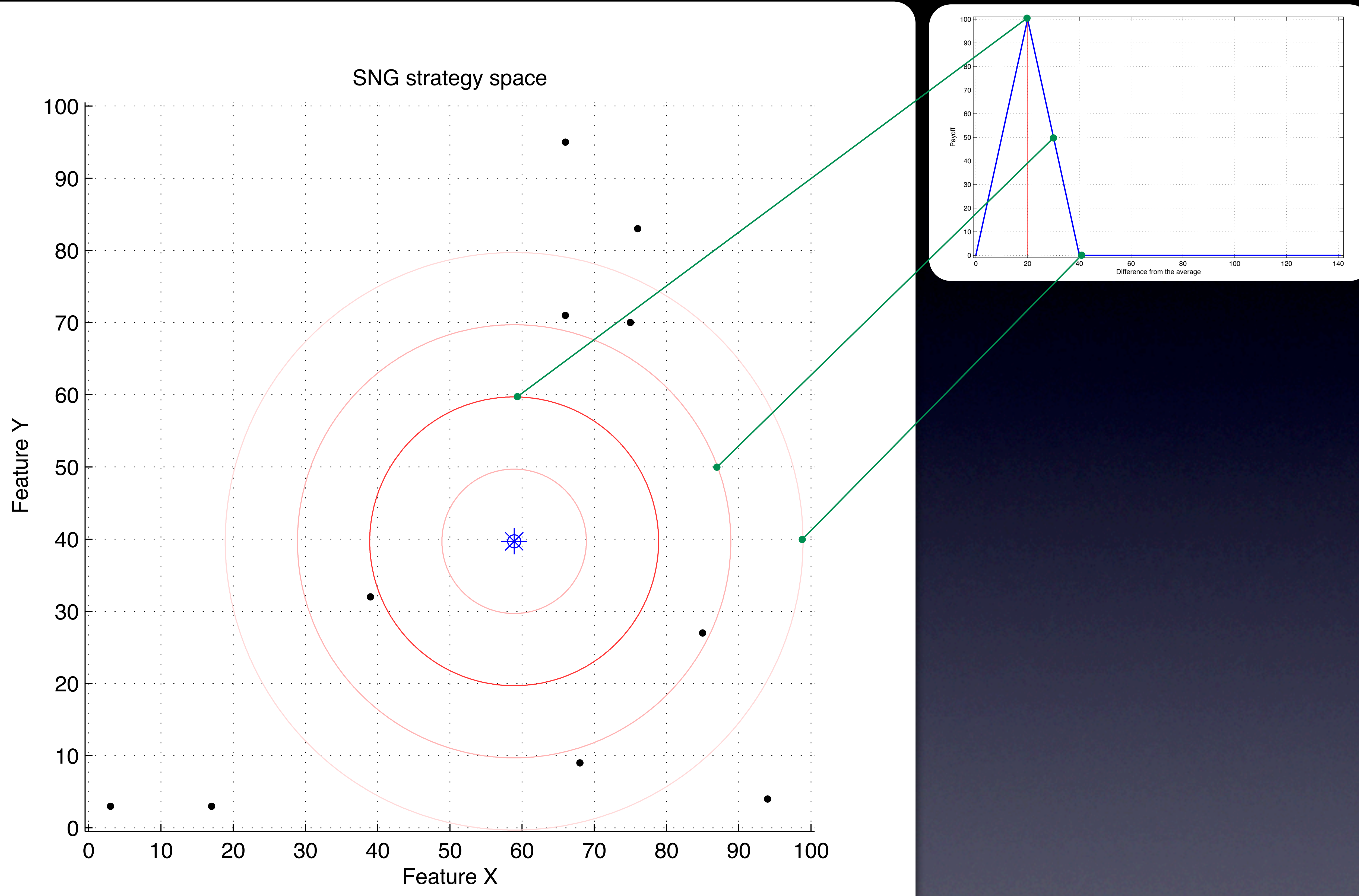

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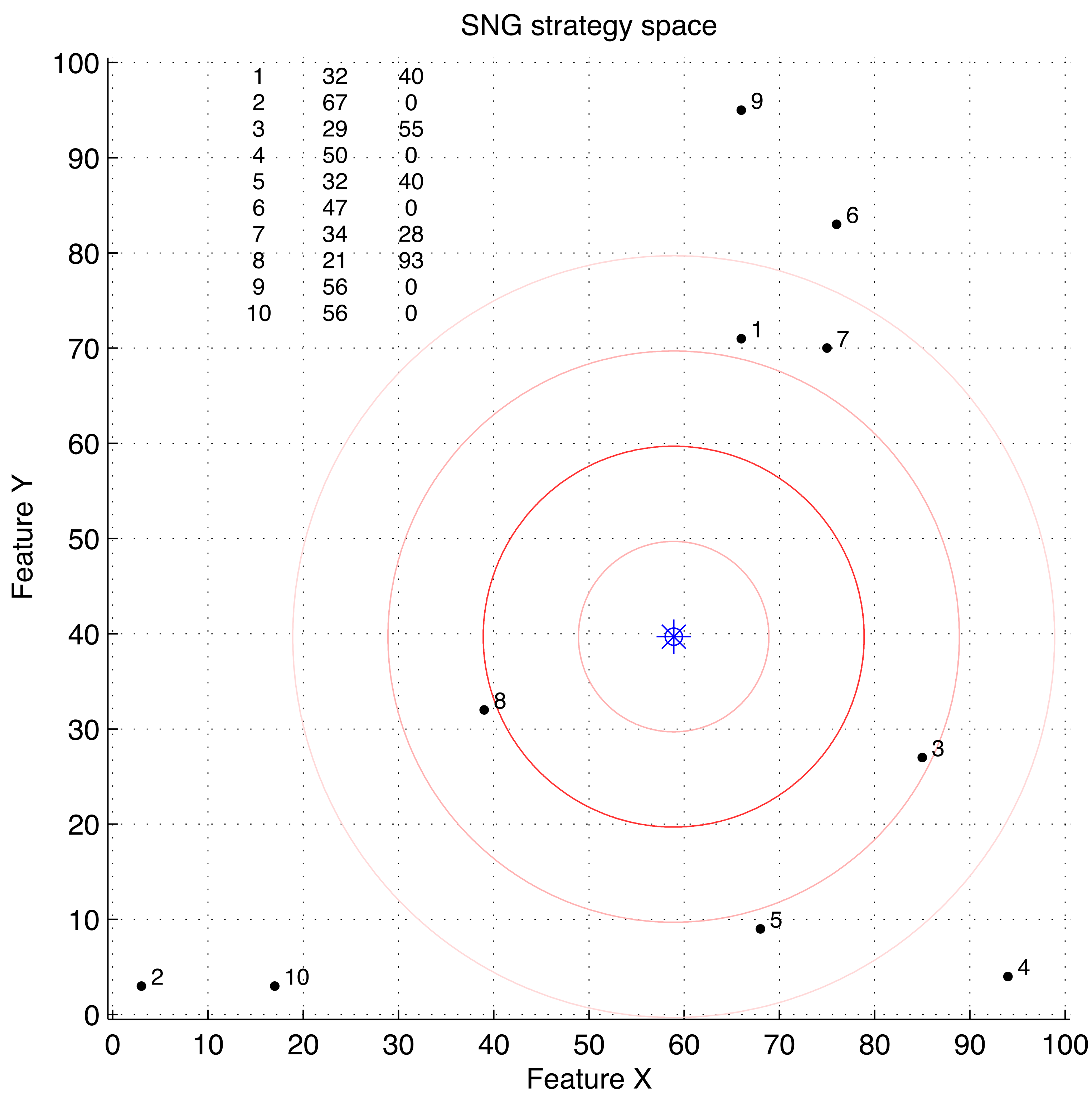

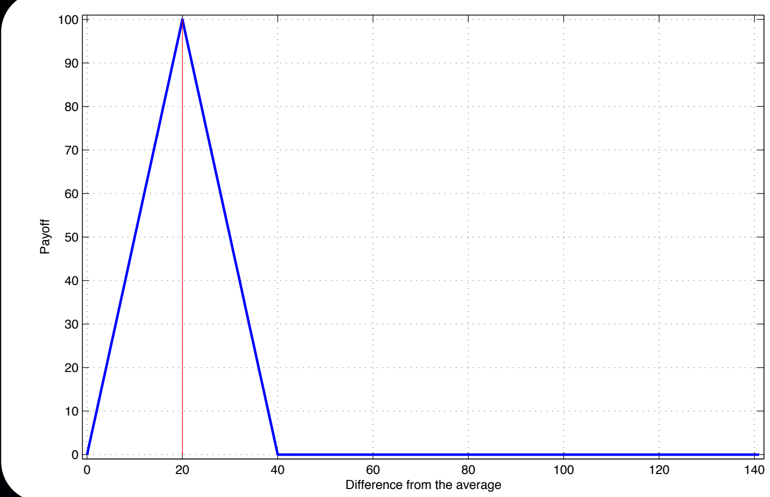

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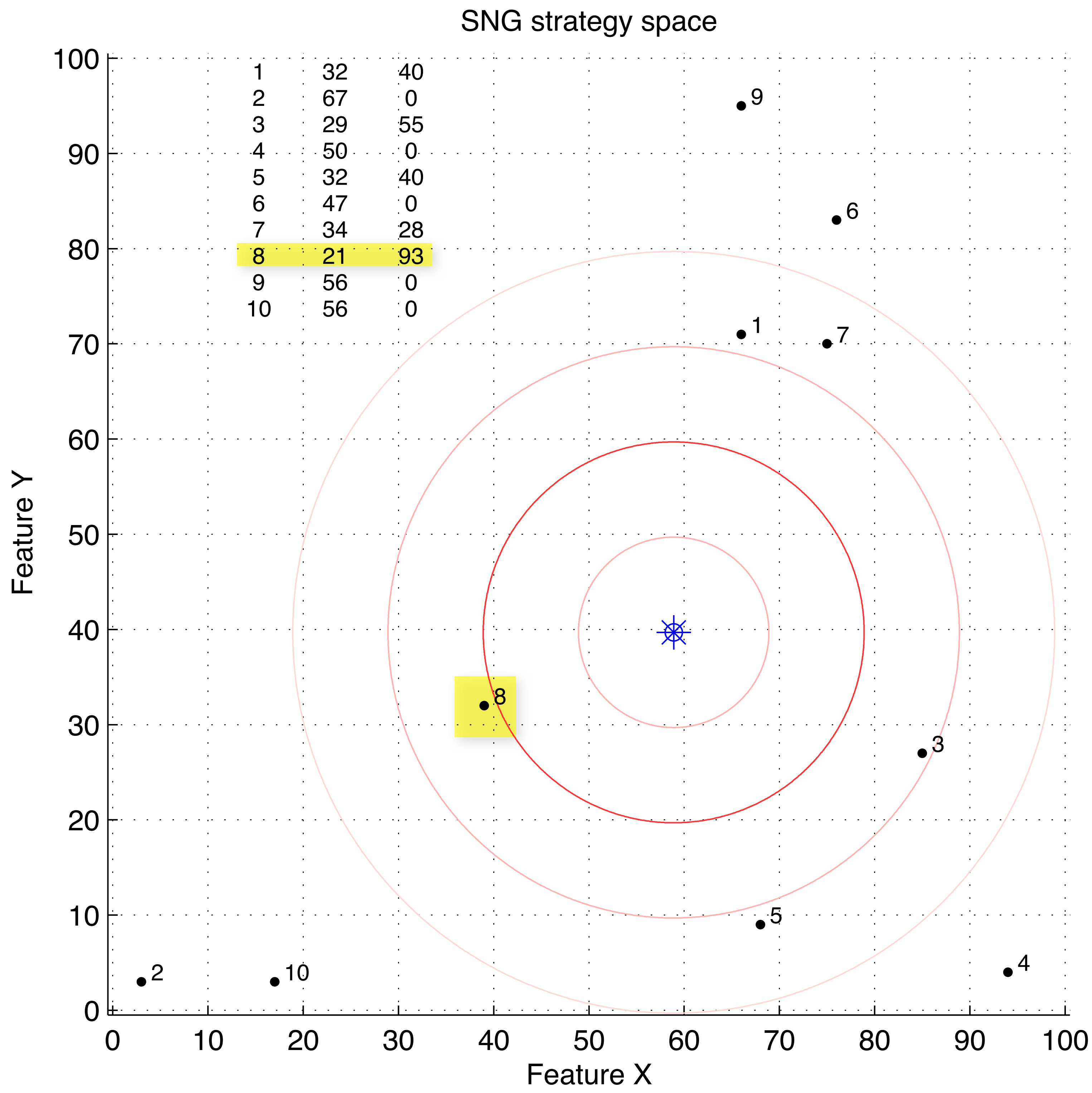

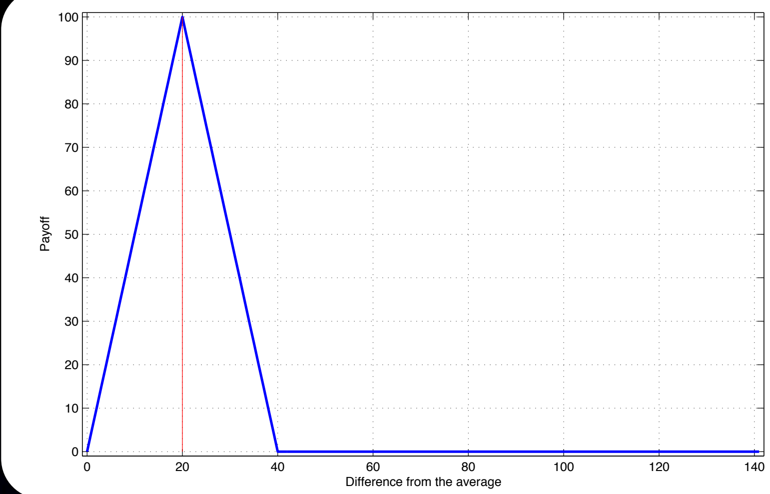

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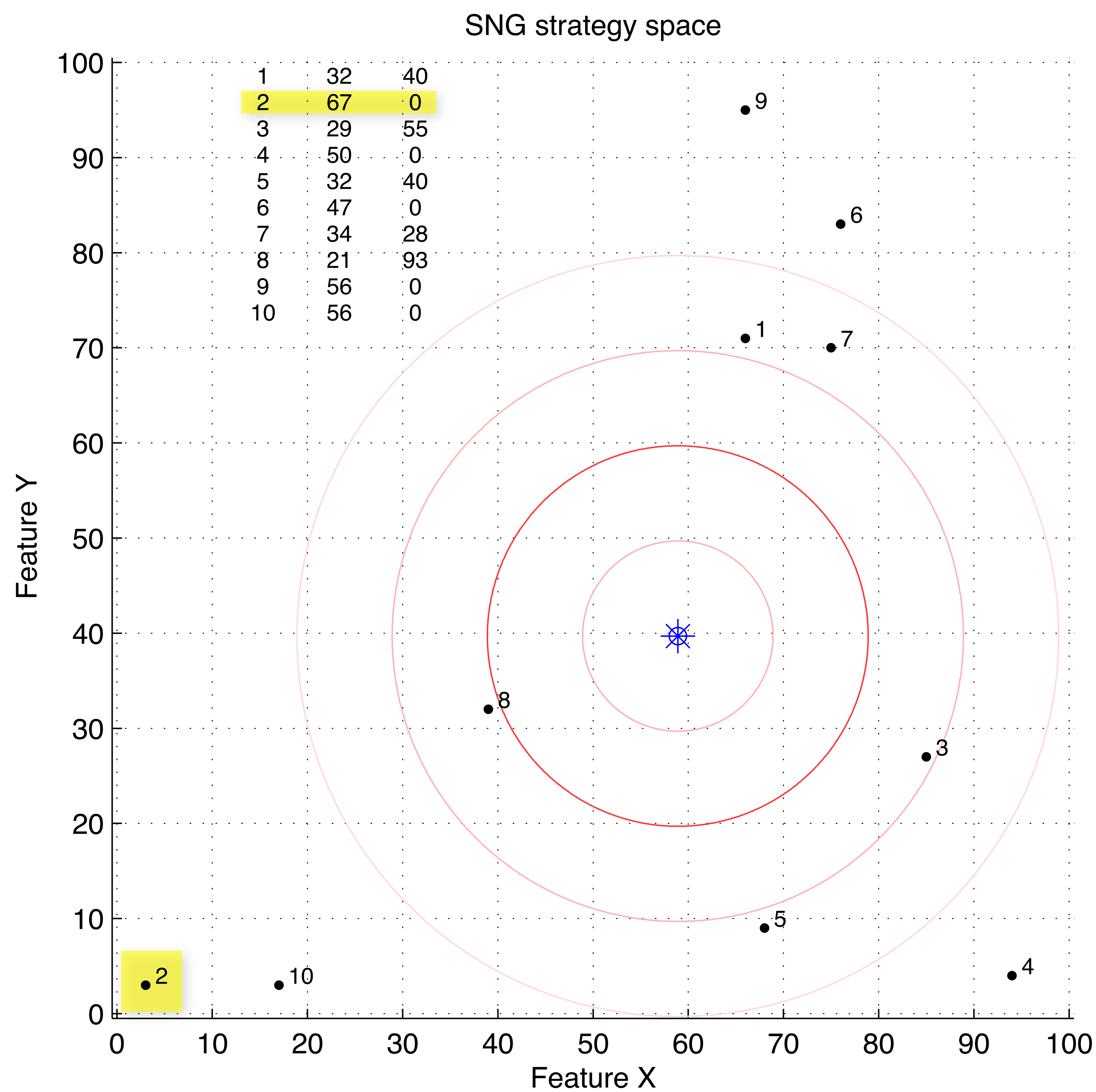

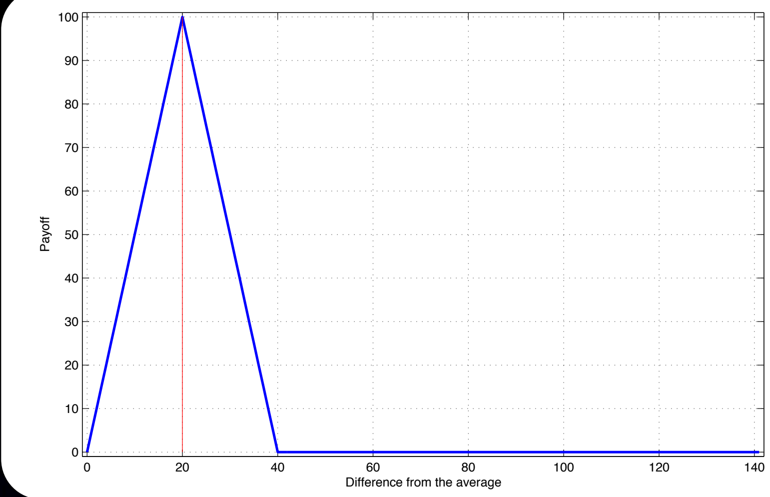

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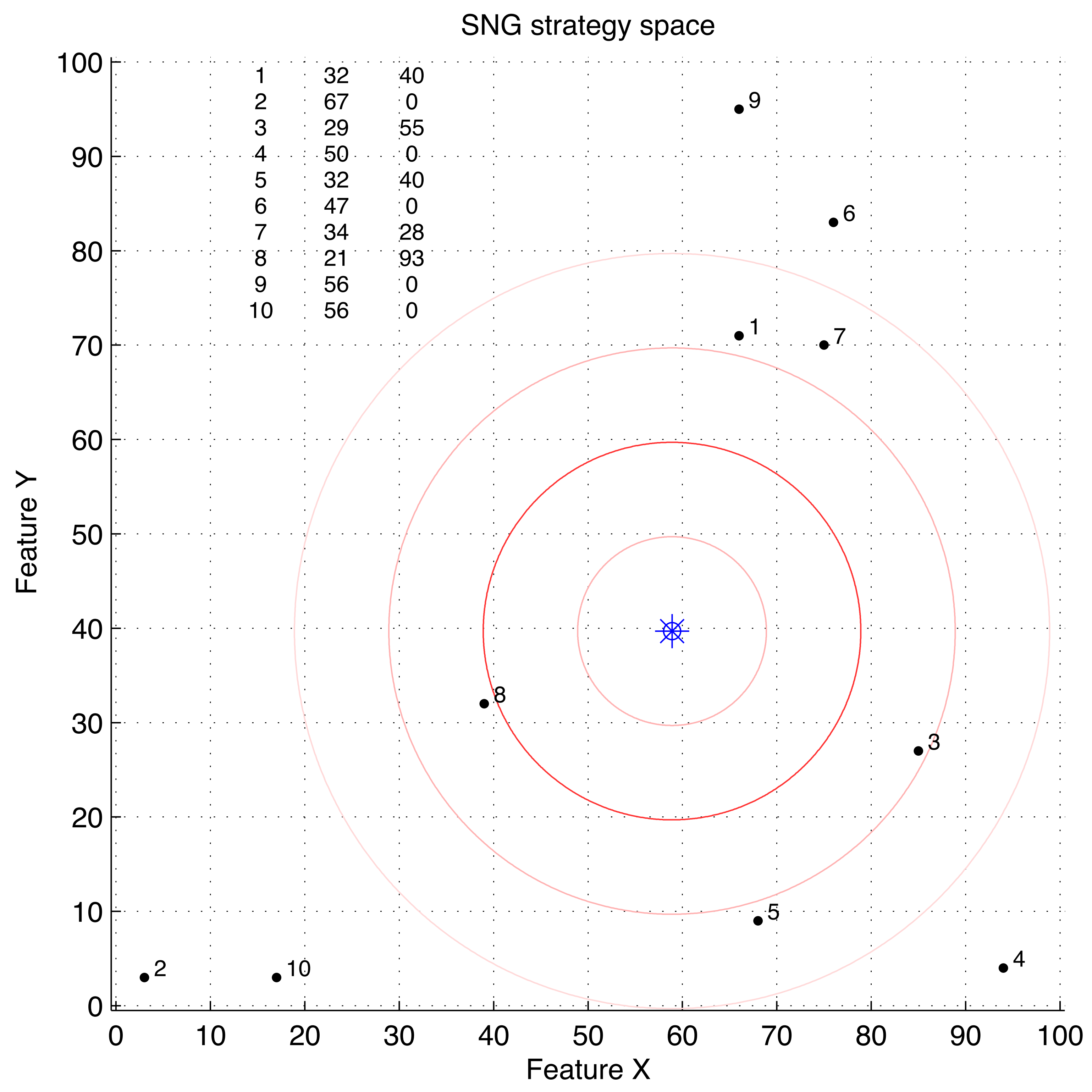

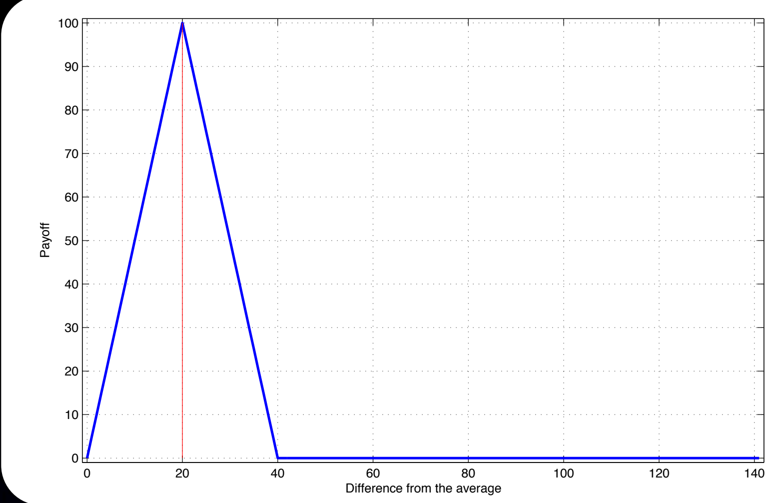

Game can be repeated over rounds

Agents have a chance to learn and adapt

Spacial Norms Game with optimal distinctiveness payoff

- Normative solution- Pure strategy Nash equilibria
  - Any set of points that are evenly distributed on the ring of distance 20 from the center point are in equilibrium
  - This is a well defined but infinitely large set
- Do simulated decision agents converge to an equilibrium set and if so how quickly?

Email Ryan for the movie files if you are interested in these.

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Spacial Norms Game with optimal distinctiveness payoff

- Simulations with simple decision agents converges to an equilibrium very quickly.
- In large part this is driven by the homogeneity of the decision agents, not their intelligence nor the simplicity of the game.
- All the decision agents are identical, which is a strong assumption.

Postulates of rationality

- A decision maker is narrowly self-interested. Her goal is to maximize personal payoffs, indifferent to other players' earnings.

Preferences

- A decision maker believes other decision makers are also narrowly self-interested.
- These qualities are common knowledge. Everyone believes that everyone believes that…∞

Beliefs

Cognitive hierarchy in games

- What if decision makers do not reason with infinite depth, but rather have limited cognitive resources?
- Further what if different decision makers have different cognitive resources?
- Formal model of distributions of types of players (Camerer, Ho, Chong, 2004) along a continuum of levels of reasoning.

Beauty Contest Game

- Each player privately selects a number between 0 an 100. It can be any real number. $\mathbb{R}$
- The person who selects the number closest to 2/3 of the mean of all the chosen numbers, earns a prize of 20 CHF, everyone else earns nothing.
- If need be, ties will be broken randomly.

Beauty Contest Game

- What is the normative solution to this game?
- If you think everyone will play randomly, then the expected mean is 50. The target is:

$$\left(\frac{2}{3} \cdot 50\right) \cong 33.33$$

- If you think everyone will reason as such, the mean would be 33.33 and 2/3 of this is...

$$\left(\frac{2}{3} \cdot 33.33\right) \cong 22.22$$

Beauty Contest Game

Levels of reasoning (k)	Expectation for the mean	Best response
0	50.00	33.33
1	33.33	22.22
2	22.22	14.81
3	14.81	9.88
4	9.88	6.58
...	...	...
Infinite	0	0

The normative solution: $50 \cdot \left(\frac{2}{3}\right)^x \qquad x \to \infty$

Beauty Contest Game

- Keynes (1936) discussed the stock market as if it were a beauty contest.
- Players would guess which contestant would get the most votes.
- The goal was not to pick the most attractive contestant, but rather the contestant that other people would think was the most attractive.

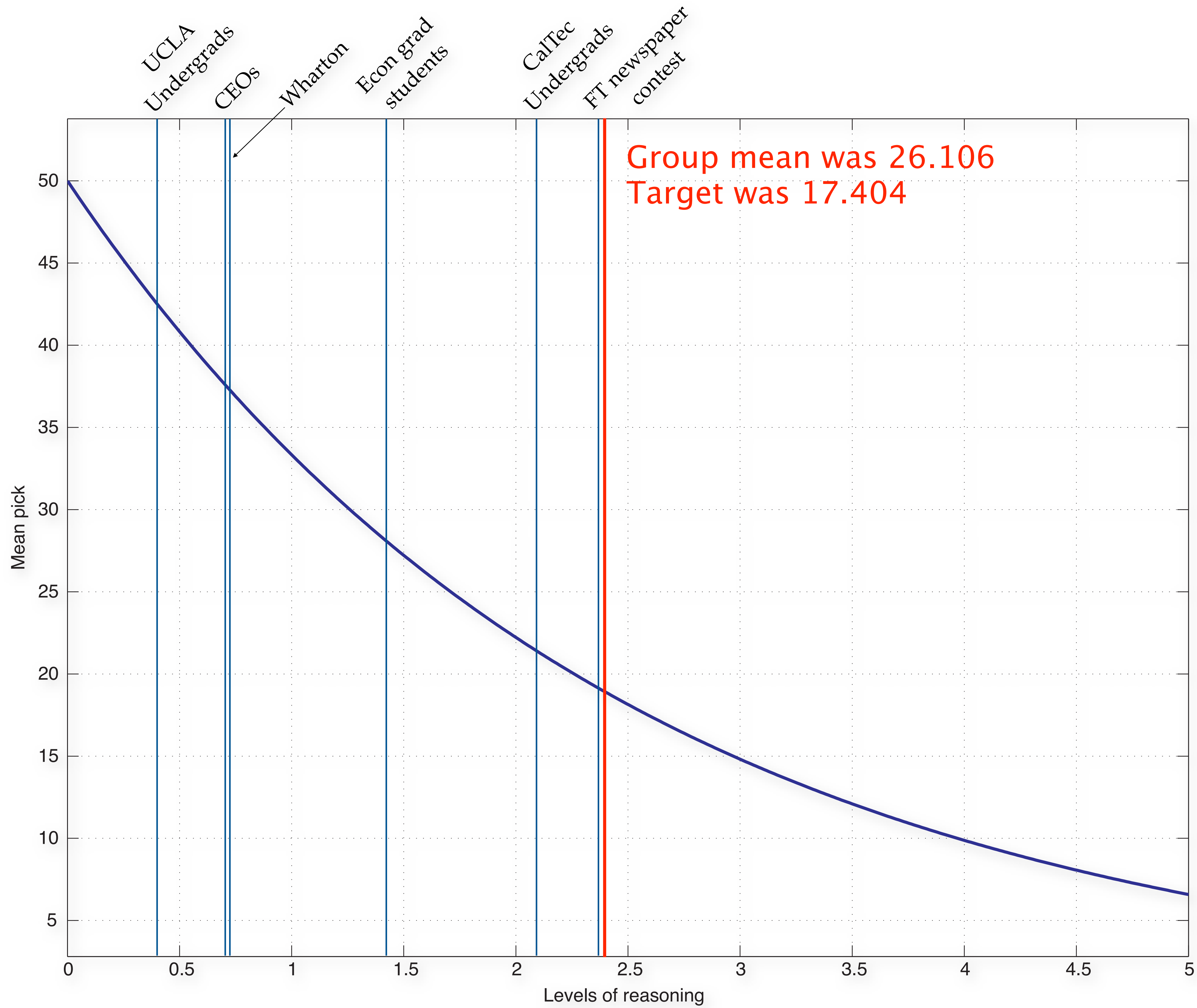
Beauty contest game results

Group mean was 26.106
Best guess was 17.404

Not 0.

0.10000	18.80000	
0.20000	19.00000	27.00000
6.70000	19.00000	30.00000
14.00000	21.00000	34.00000
14.00000	22.00000	36.00000
14.00000	22.20000	42.00000
15.00000	22.40000	93.00000
16.60000	24.00000	99.00000
18.23300	24.42000	

ETH data 2011



ETH data 2011

Cognitive hierarchy in games

- Behavioral game theory seeks to extend normative assumptions with more realistic and descriptively accurate postulates regarding interacting decision makers
- Heterogeneity in types with varying degrees of cognitive sophistication (bounded rationality)
- Note that this deviation from perfect rationality is not just rationality plus noise, but a well defined formulation with a clear interpretation

Cognitive hierarchy in games

- Consider a K1 agent
  - K1 agents presume that all other decision agents are static and non-adaptive. These K1 agents best respond to the current environment. These are one-step reasoning players.
- Consider a K2 agent
  - K2 agents presume that all other decision agents are K1. These K2 agents best respond to what they anticipate the other K1 agents will do.

Too smart for their own good

The impact of K2 level reasoning

- Beyond a fraction of ~ 50 % K2 level reasoning players, the average payoff drops significantly
- Results are robust for a wide range of models varying exogenous noise

Too smart for their own good

The impact of K2 level reasoning

- K2 level players belief that others are playing simple one-step best response (K1)
- Beyond a critical fraction this wrong belief leads to suboptimal choices of K2 level players and diminishes payoff for all players

Too smart for their own good

The impact of K2 level reasoning

- Above the critical fraction of K2s, ‘cyclic’ dynamics emerge in the strategy space
- K2 level players anticipate and compensate for the ‘trend’ in the center point
- Wrong beliefs about others let players underestimate the change in strategy space

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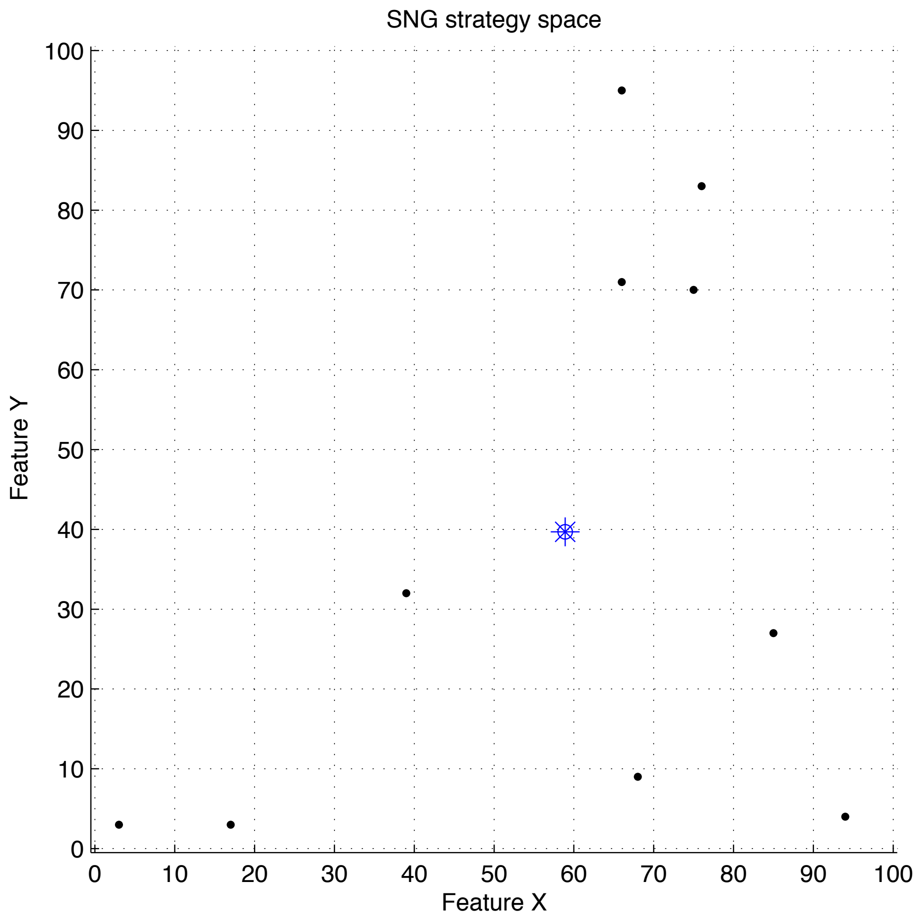
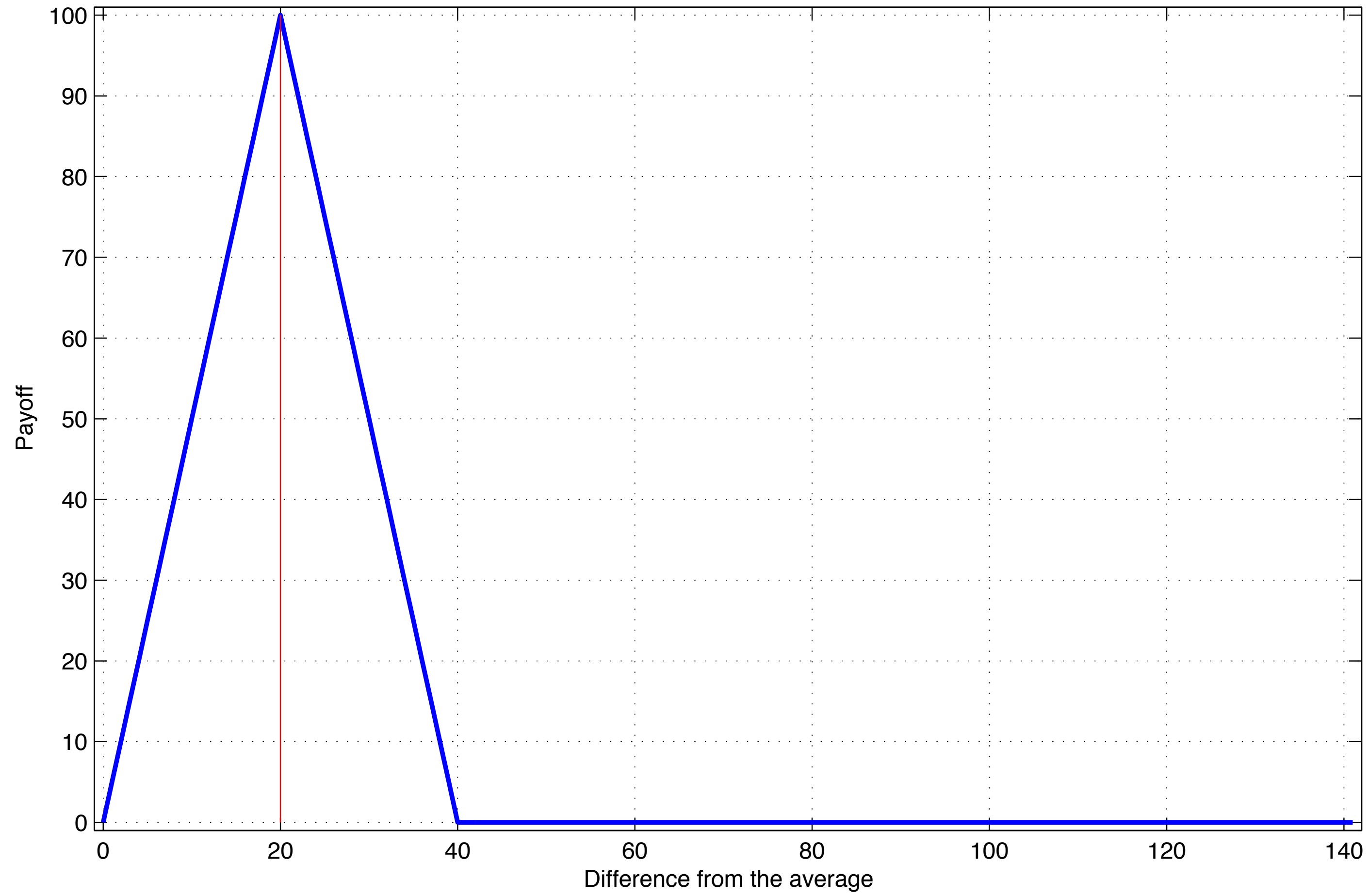
Changing the payoff function

- The SNG is a general framework useful for studying partial coordination decision problems.
- The particular game is defined in part by the payoff function that is used.
- Different payoff functions lead to different games and different dynamics. Optimal distinctiveness is just one possible payoff function.

SNG framework



Optimal distinctiveness payoff function



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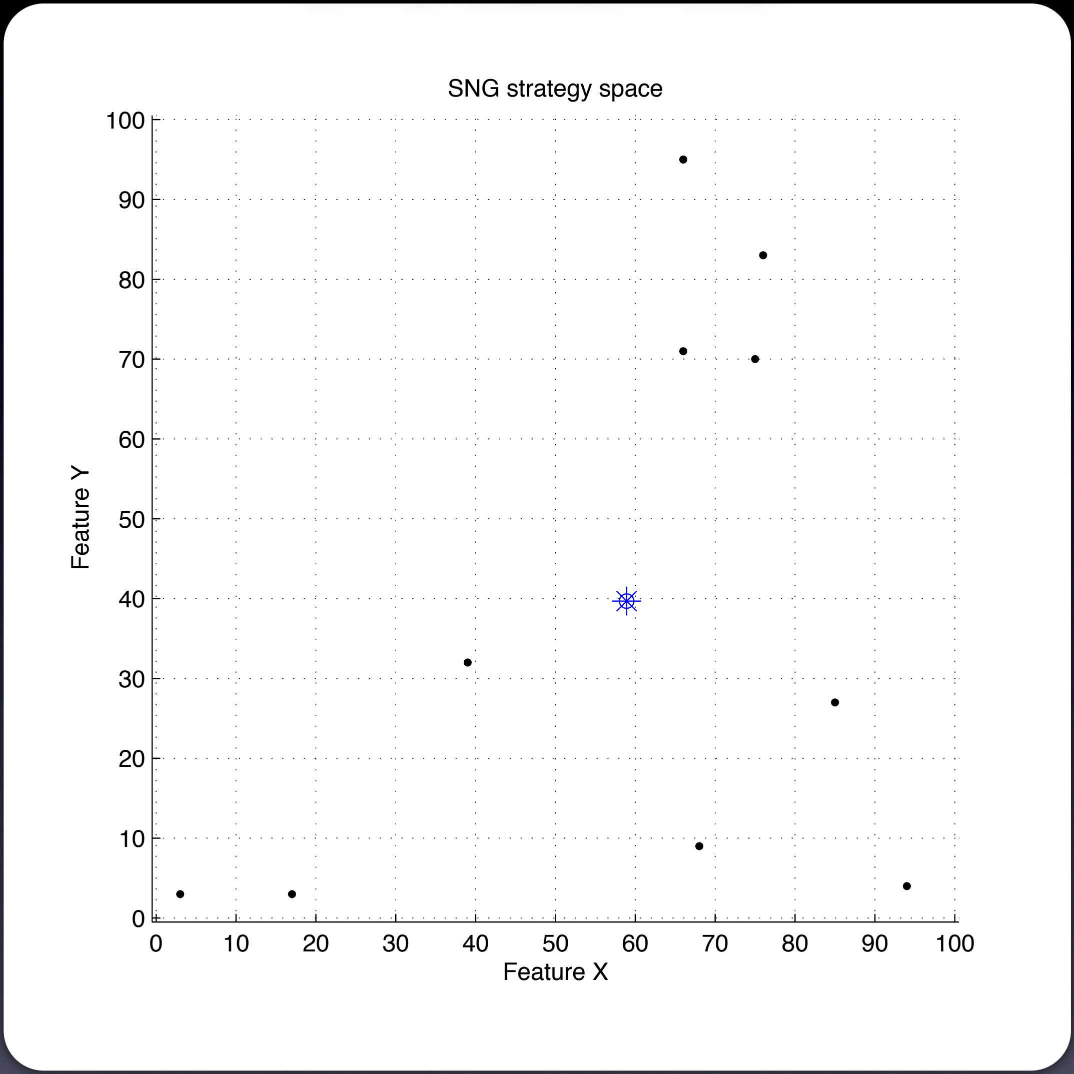
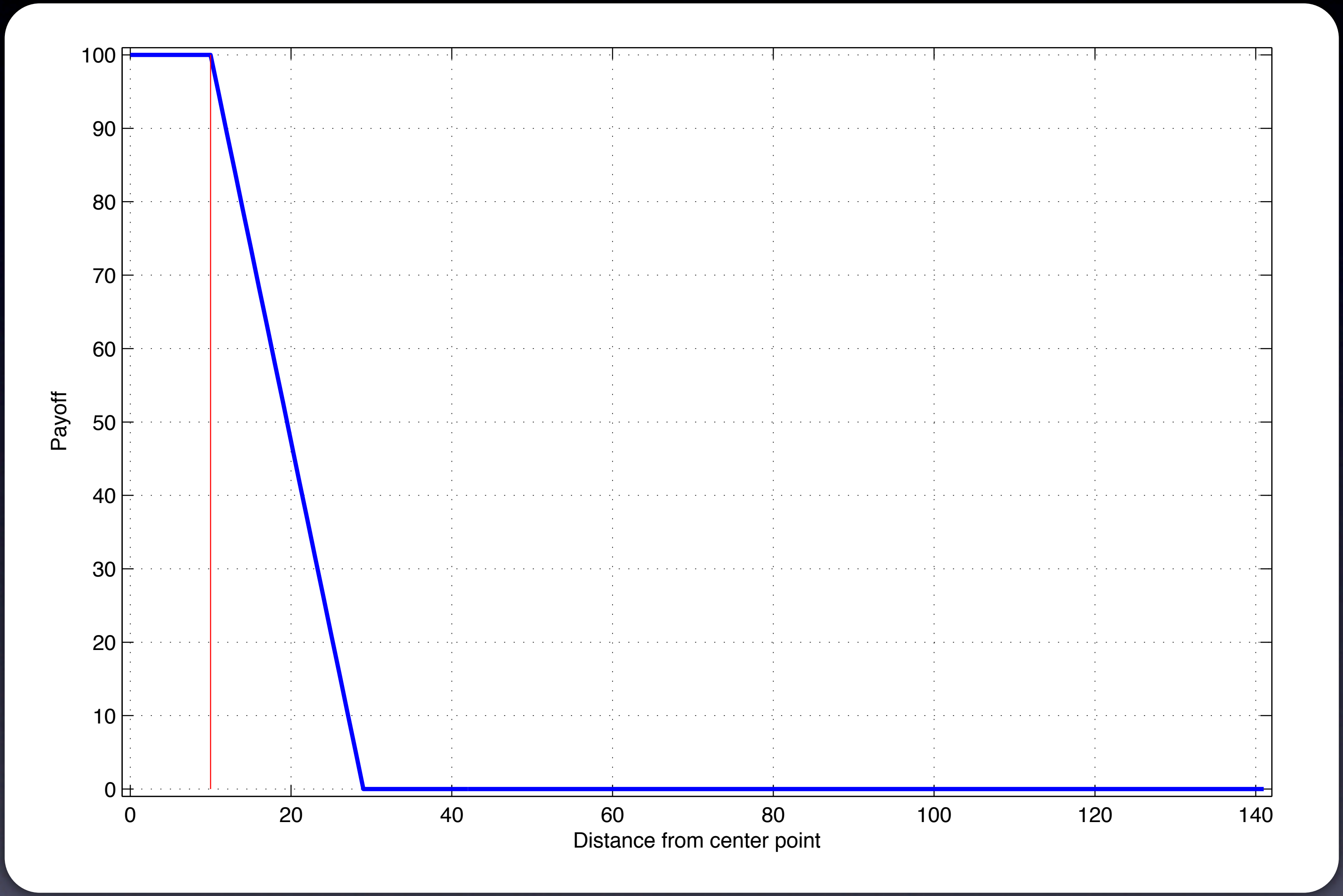
SNG framework

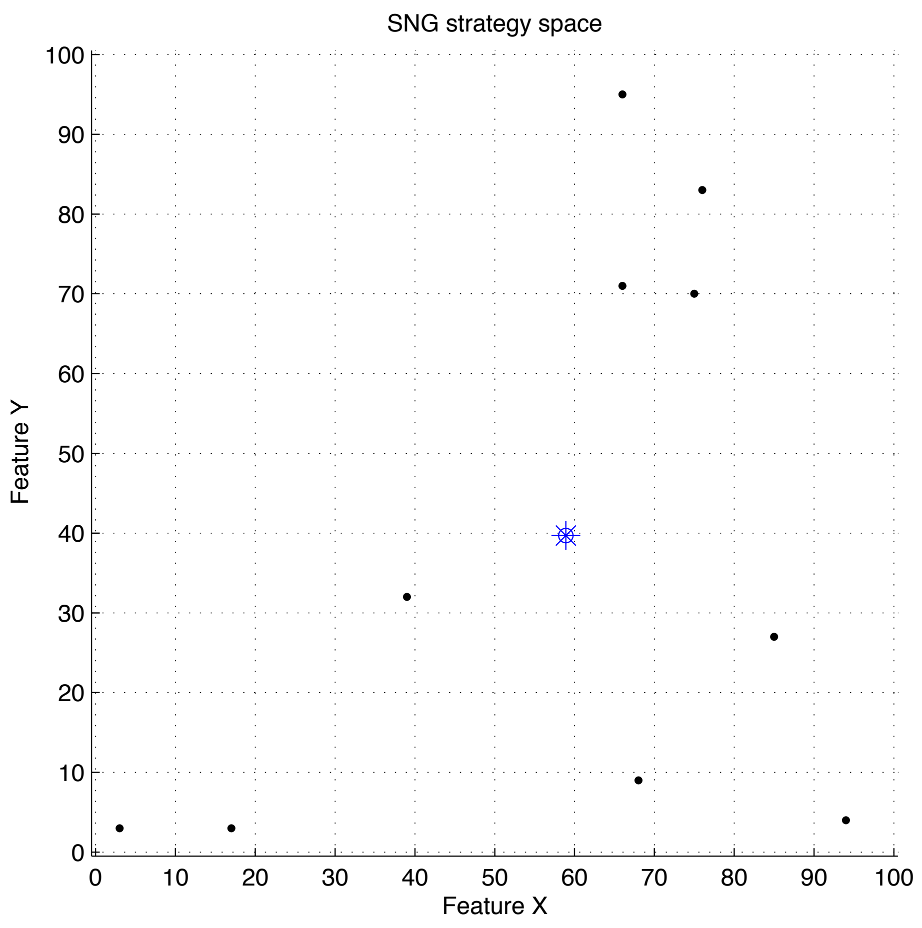


Easy coordination payoff function

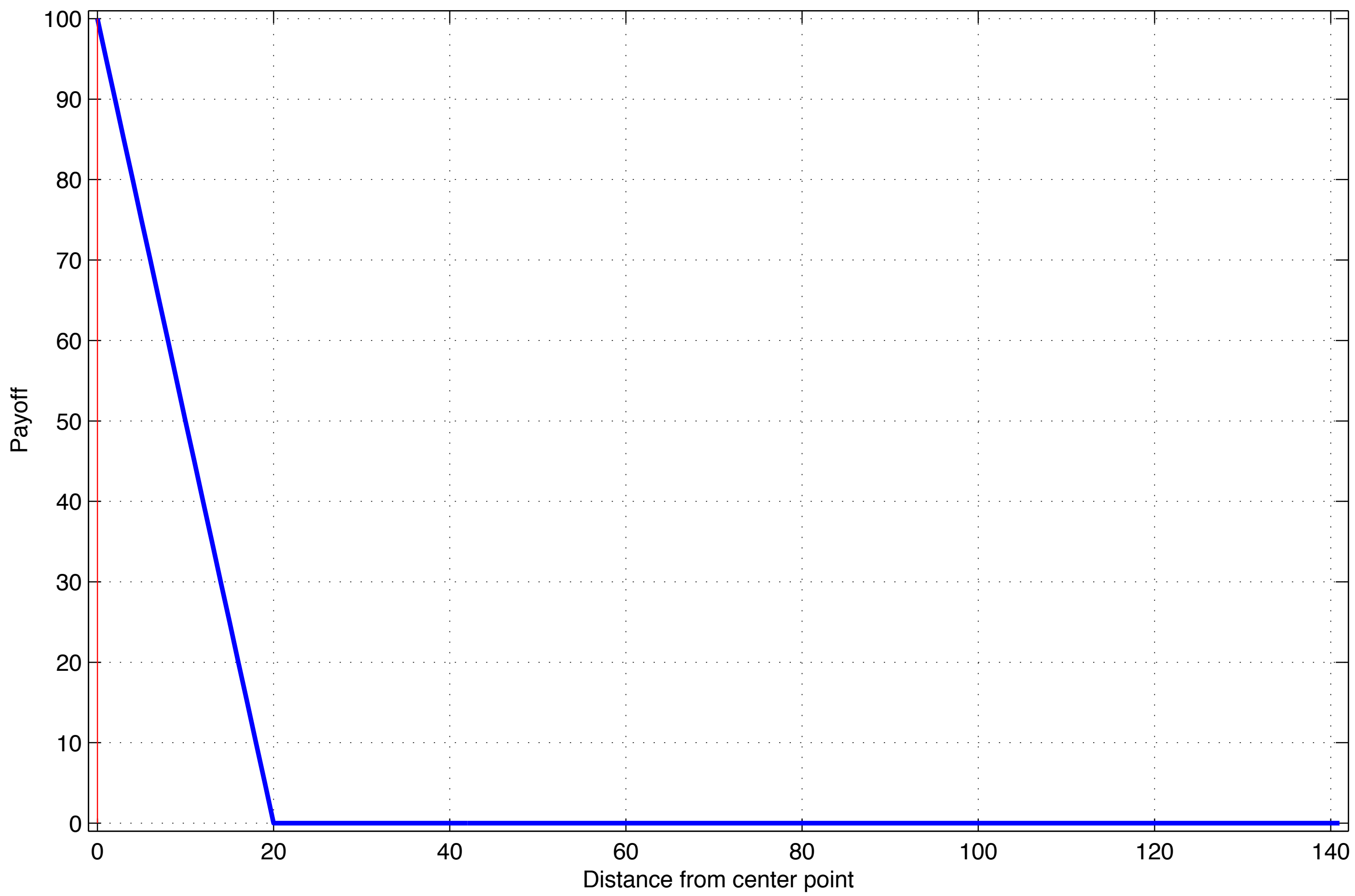


Modularity

SNG framework

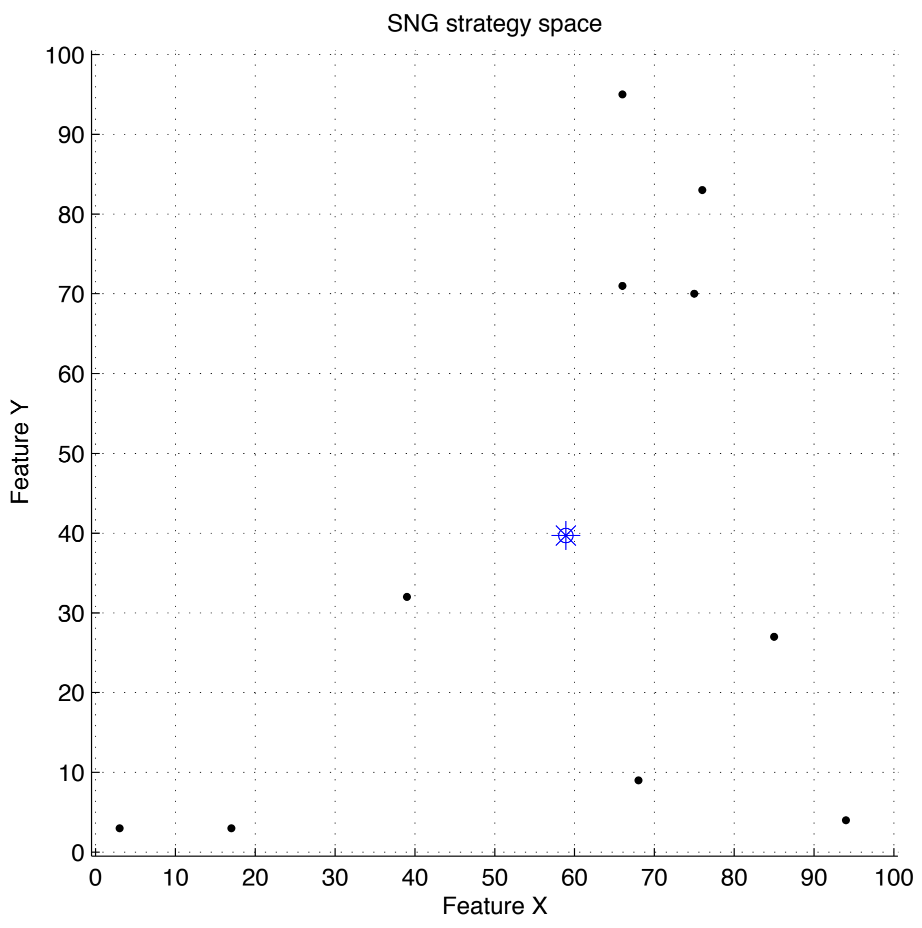


Point coordination payoff function

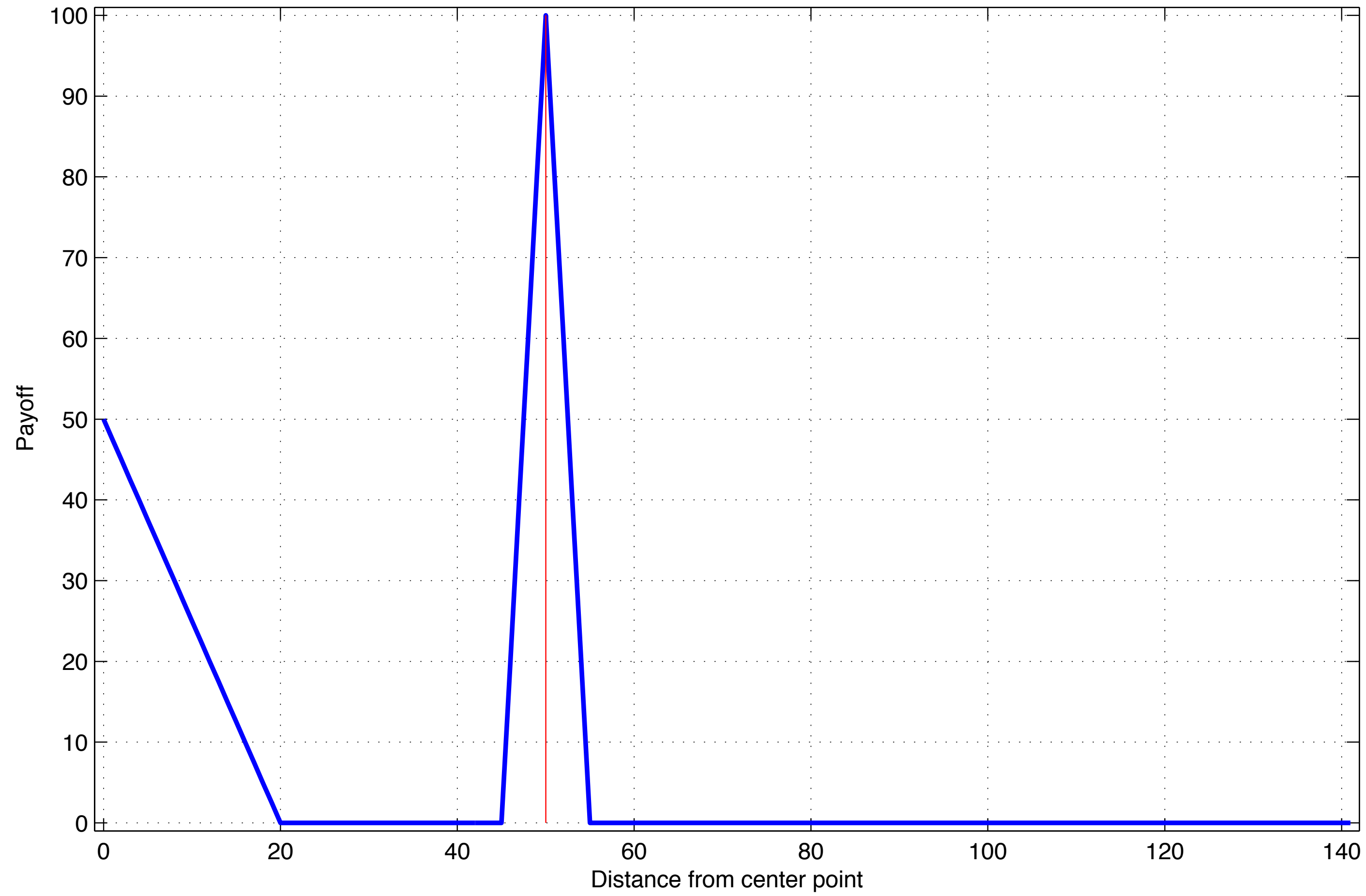


Modularity

SNG framework

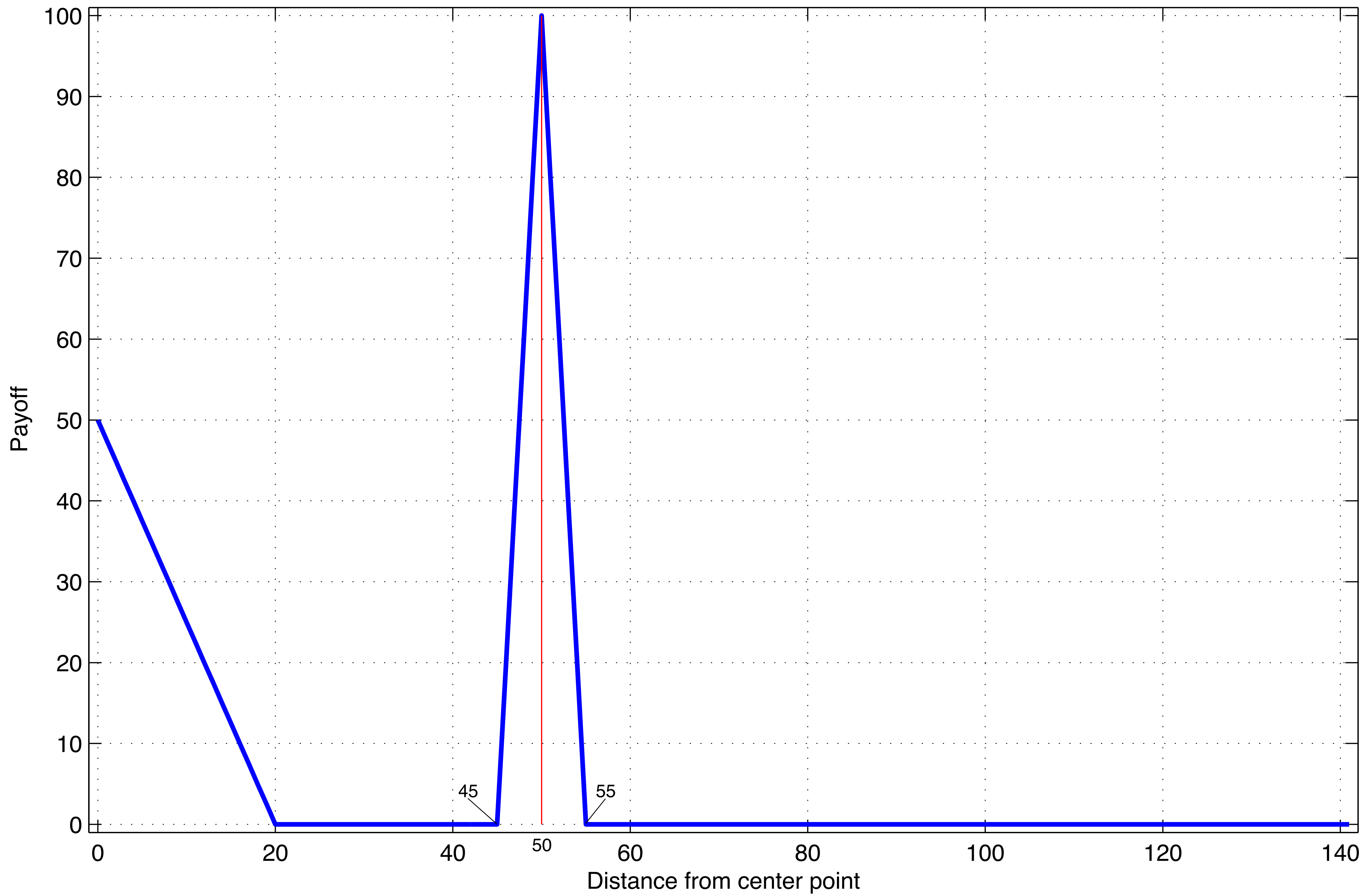


Innovation payoff function



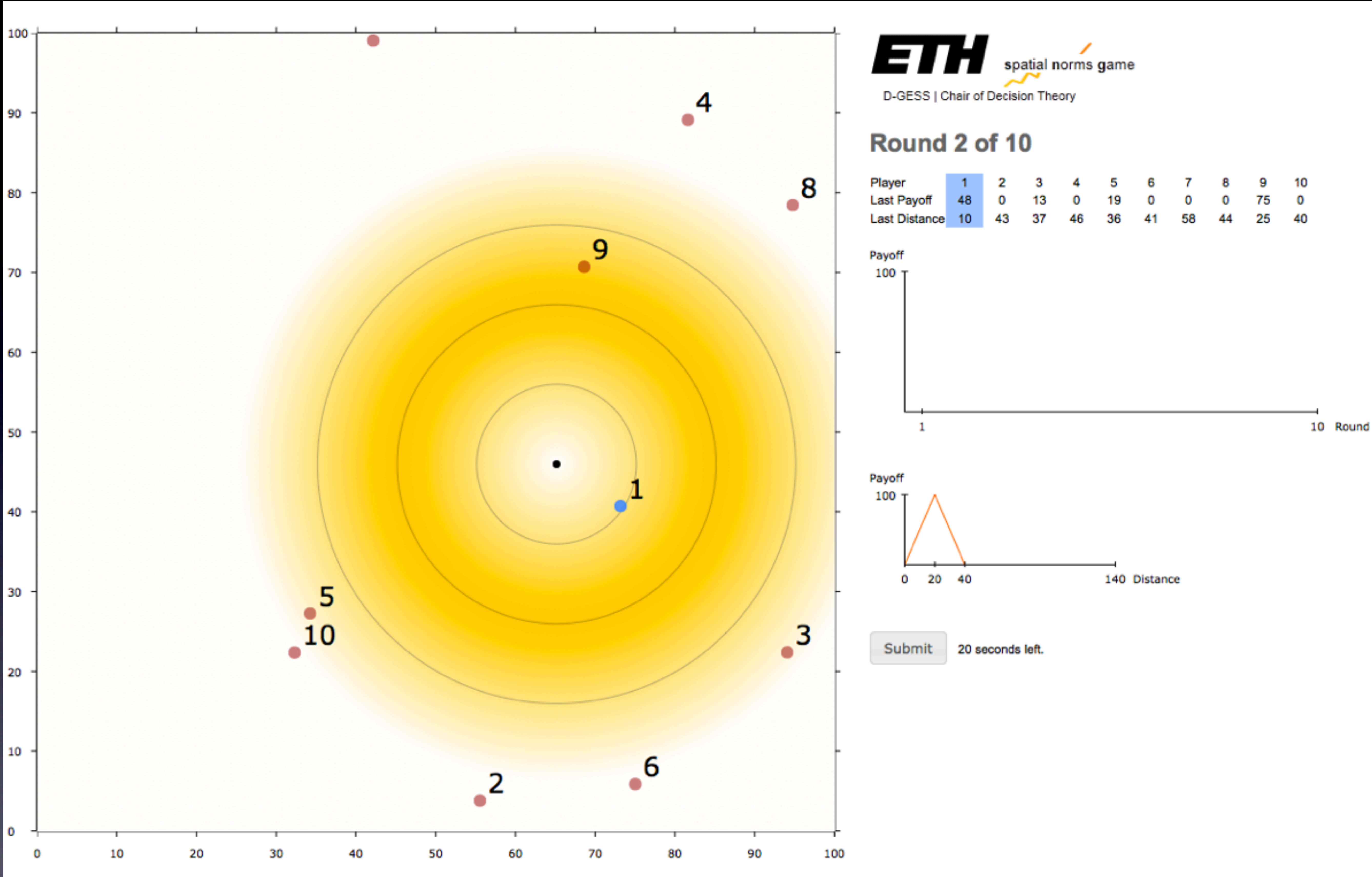
Modularity

Innovation payoff function



Other variations and extensions

- Including different groups of players is possible. In these extensions, the decision agents have an incentive to be optimally distinct from their in-group members AND maximally distinct from out-group members.
- Kurt Ackerman (2011) has a set of simulation results related to these cases.



Programed version for experiments (alpha v0.3) http://vlab.ethz.ch/normsgame/

- A mathematical model of a norms game where many interacting decision agents have interdependent payoffs
- Simple dynamics emerge with homogeneous one-step learning agents
- Complex dynamics emerge with the interactions of heterogeneous agents with more cognitively sophisticated decision agents

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